EE1 and ISE1 Communications I

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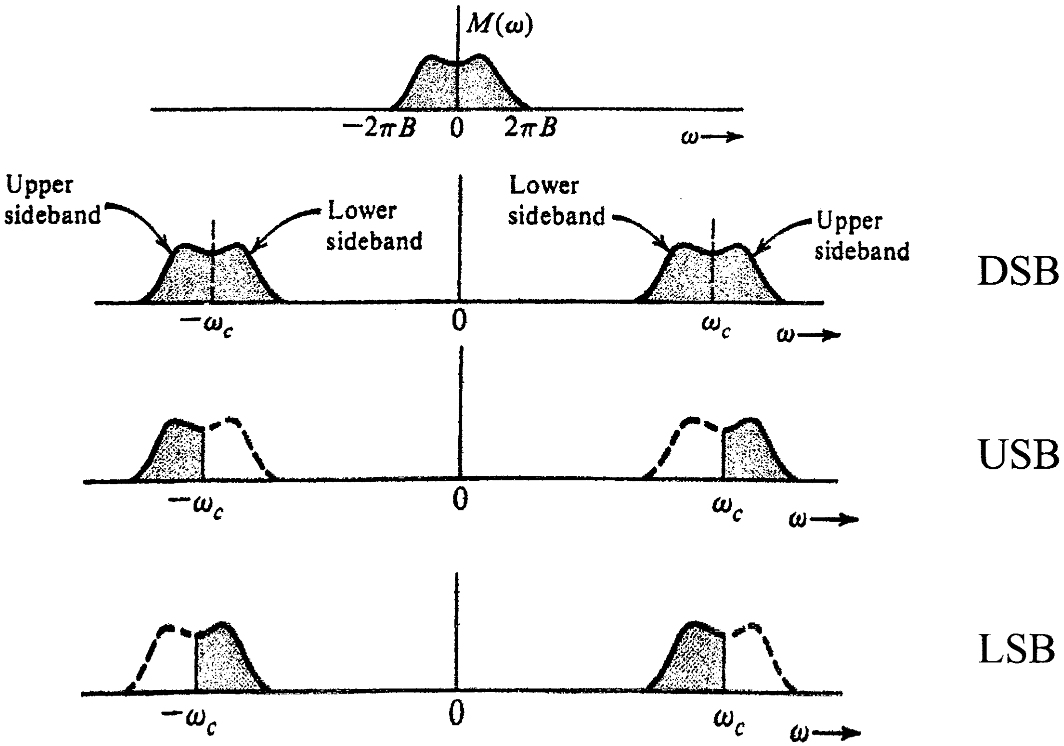
Lecture eleven

Lecture Aims

- To examine Single Sideband Modulation (SSB)
  - Time domain representation
  - Tone modulation
  - Generation of SSB signals
  - Demodulation of SSB signals

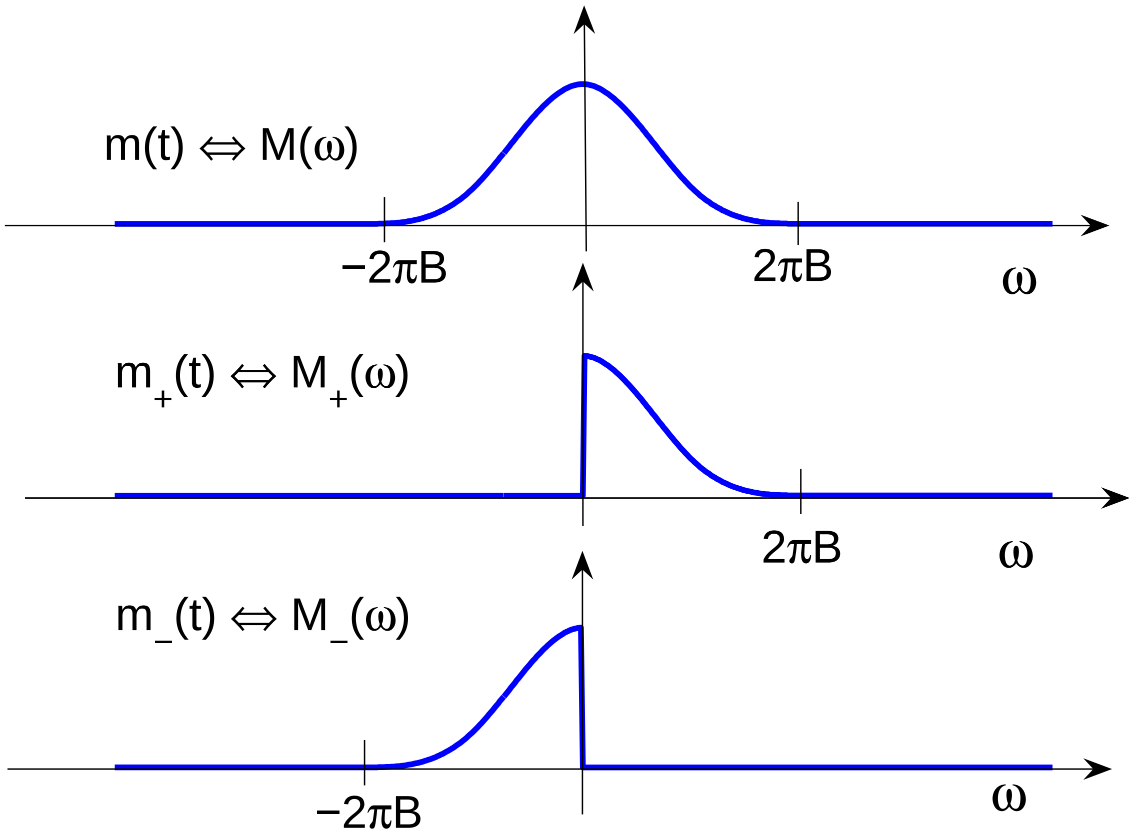
Modulation of Baseband Signals

Modulated Signal



Modulation of Baseband Signals

Splitting the baseband spectrum into USB and LSB



Single Sideband Generation

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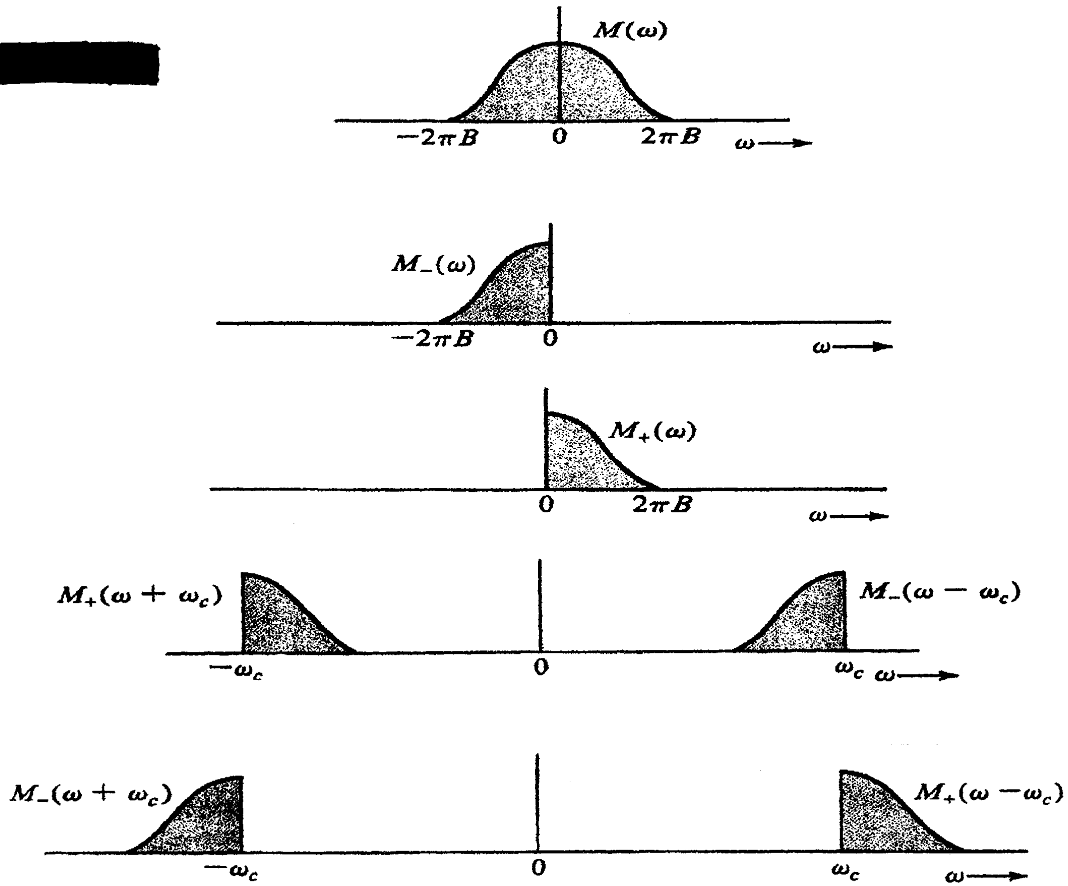


Time-Domain Representation of SSB signals

- Let $m_+(t)$ and $m_-(t)$ be the inverse Fourier transforms of $M_+(\omega)$ and $M_-(\omega)$.
- Because the amplitude spectra $|M_+(\omega)|$ and $|M_-(\omega)|$ are not even functions of ω, the signals $m_+(t)$ and $m_-(t)$ cannot be real. They are complex.
- It can be proven that $m_+(t)$ and $m_-(t)$ are conjugates. Moreover, $m_+(t) + m_-(t) = m(t)$. Hence,

$$m_+(t) = \tfrac{1}{2}[m(t) + jm_h(t)]$$

$$m_-(t) = \tfrac{1}{2}[m(t) - jm_h(t)]$$

Time-Domain Representation of SSB signals

To determine $m_h(t)$ note that

$$\begin{aligned} M_+(\omega) &= M(\omega)u(\omega) \\ &= \tfrac{1}{2}M(\omega)[1+\mathsf{sgn}(\omega)] \\ &= \tfrac{1}{2}M(\omega)+\tfrac{1}{2}M(\omega)\mathsf{sgn}(\omega) \end{aligned}$$

Since $m_+(t) = \frac{1}{2}[m(t) + jm_h(t)]$, it follows $jm_h(t) \Longleftrightarrow M(\omega)\mathsf{sgn}(\omega)$. Hence $M_h(\omega) = -jM(\omega)\mathsf{sgn}(\omega)$. But $1/\pi t \Longleftrightarrow -j\mathsf{sgn}(\omega)$. Therefore

$$m_h(t) = m(t) * 1/\pi t = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha} d\alpha.$$

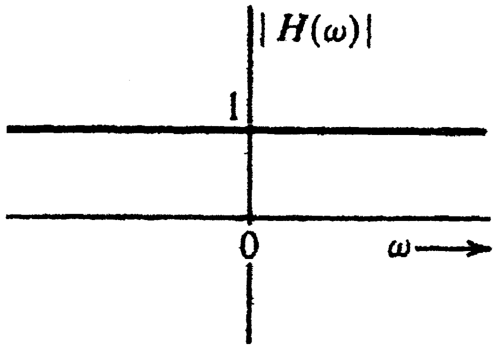
The right-hand side of this last equation defines the **Hilbert transform** of $m(t)$.

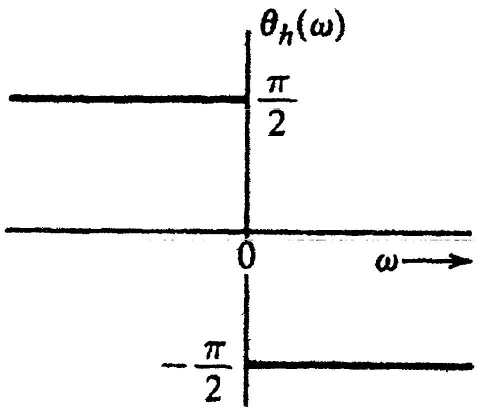
Hilbert Transform

The Hilbert Transform $m_h(t)$ is generated by passing $m(t)$ through a filter $h(t)$ with the following transfer function:

$$H(\omega) = -j\text{sgn}(\omega) = \begin{cases} -j = e^{-j\pi/2} & \omega \geq 0 \\ j = e^{j\pi/2} & \omega < 0 \end{cases}$$

That is, $|H(\omega)| = 1$ and $\theta_h(\omega) = -\pi/2$, for $\omega \geq 0$





Time-Domain Representation of SSB Signals

We can now express the SSB signal in terms of $m(t)$ and $m_h(t)$.

$$\Phi_{USB}(\omega) = M_+(\omega - \omega_c) + M_-(\omega + \omega_c)$$

Inverse transform gives

$$\phi_{USB}(t) = m_+(t)e^{j\omega_c t} + m_-(t)e^{-j\omega_c t}$$

Using

$$m_+(t) = \tfrac{1}{2}[m(t) + jm_h(t)]$$

$$m_-(t) = \tfrac{1}{2}[m(t) - jm_h(t)]$$

$$\phi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

Time-Domain Representation of SSB Signals

In a similar way we can show that

$$\phi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t.$$

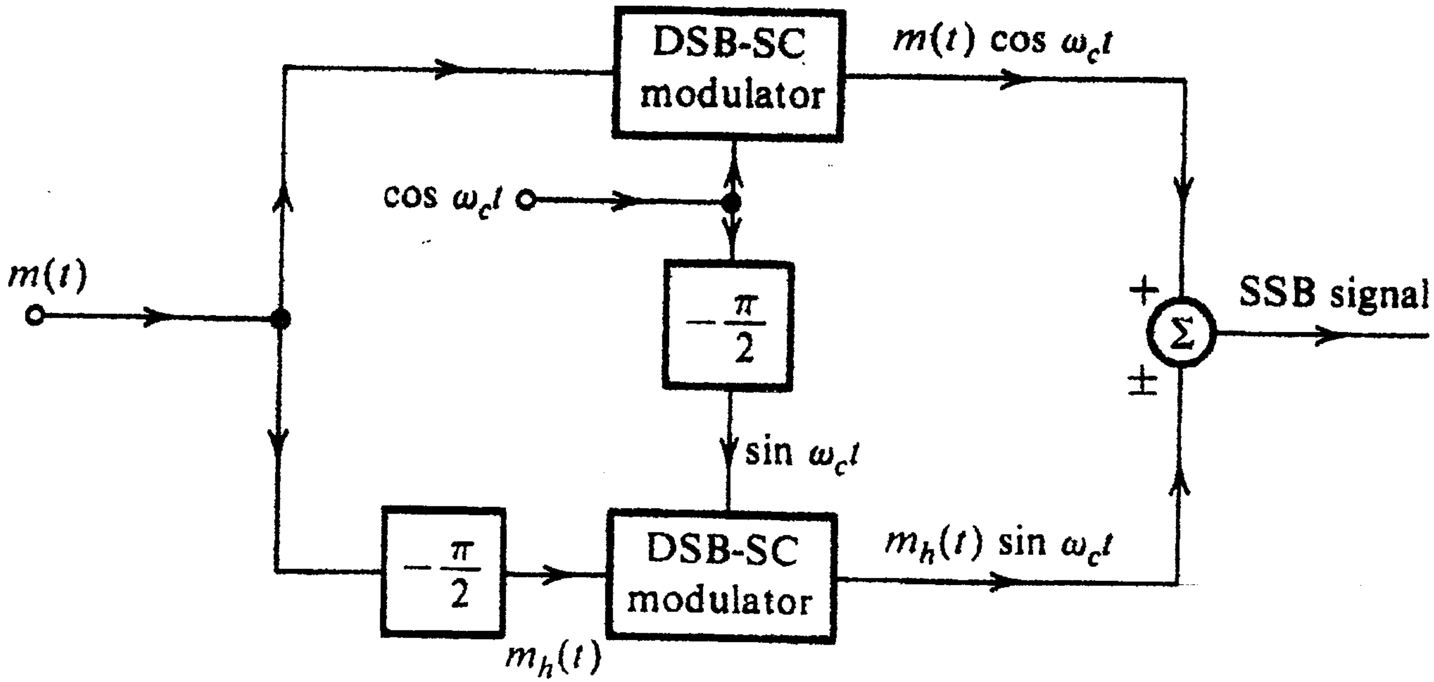
Hence a general SSB signal $\phi_{SSB}(t)$ can be expressed as

$$\phi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t,$$

where the minus sign applies to USB and the plus sign applies to LSB.

Generation of SSB Signals

Phase-Shift Method: $\phi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$



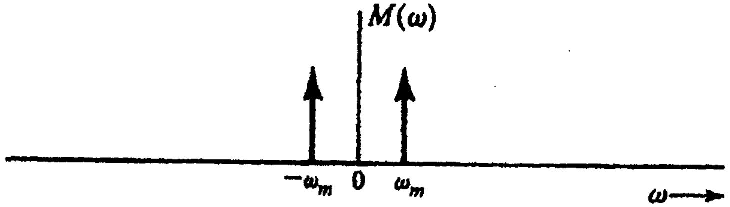
SSB Tone Modulation Example

- Consider single tone modulating signal: $m(t) = \cos \omega_m t$.
- Hilbert transform requires phase shift by $\pi/2$.
- Delay in phase by $\pi/2$ yields $m_h(t) = \cos(\omega_m t - \pi/2) = \sin \omega_m t$.
- Using $\phi_{SSB}(t) = m(t) \cos \omega_c t \mp m_h(t) \sin \omega_c t$, we get

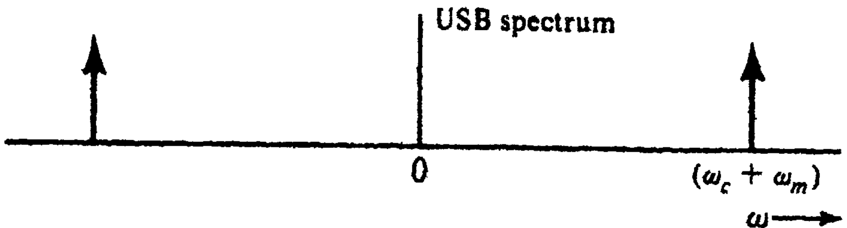
$$\phi_{SSB}(t) = \cos \omega_m t \cos \omega_c t \mp \sin \omega_m t \sin \omega_c t = \cos(\omega_c \pm \omega_m)t.$$

SSB Tone Modulation Example

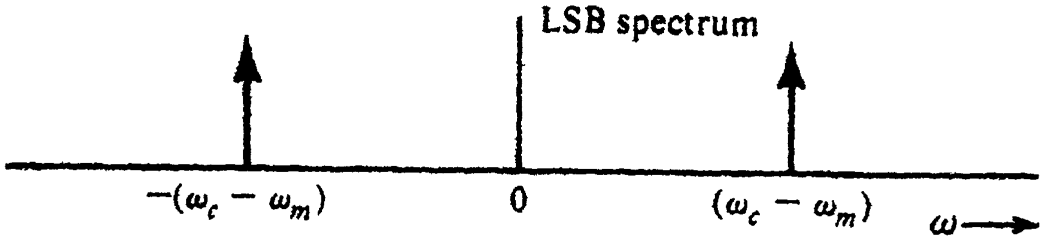
- **Baseband spectrum**



- **USB spectrum**

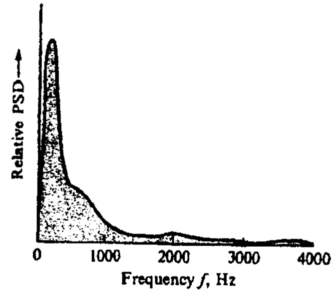


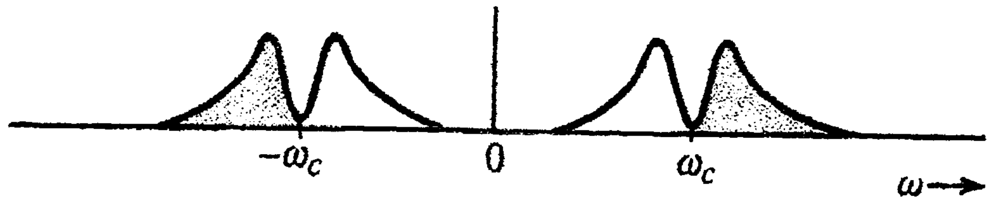
- **LSB spectrum**

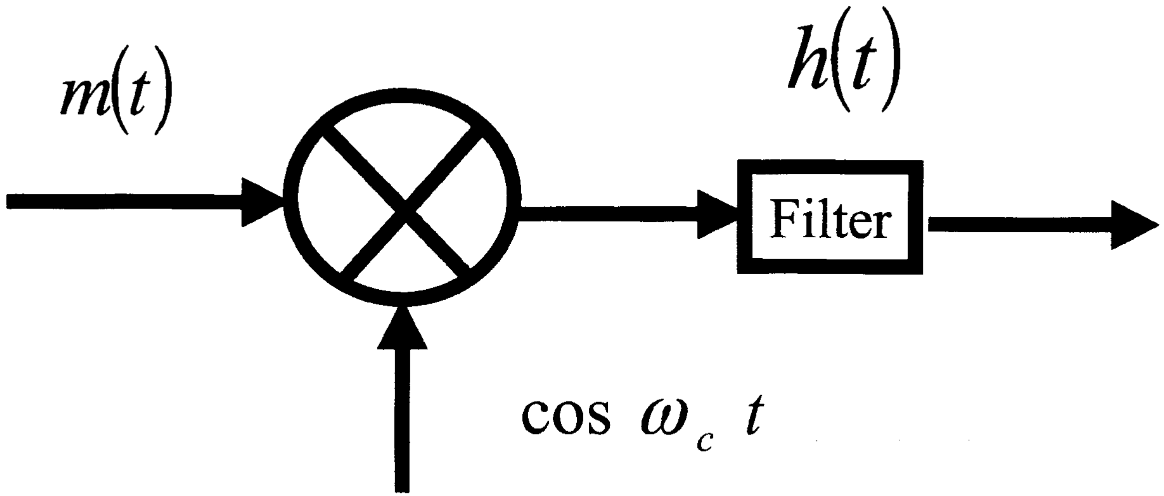


Generation of SSB Signals

Selective-filtering method:







Coherent demodulation of SSB-SC signals

The SSB demodulator is identical to the synchronous demodulator used for DSB-SC.

$$\phi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$$

Hence

$$\begin{aligned}\phi_{SSB}(t)\cos(\omega_c t) &= \tfrac{1}{2}m(t)[1+\cos 2\omega_c t] \mp m_h(t)\sin 2\omega_c t \\ &= \tfrac{1}{2}m(t) + \tfrac{1}{2}[m(t)\cos 2\omega_c t \mp m_h(t)\sin 2\omega_c t]\end{aligned}$$

Thus, the product $\phi_{SSB}(\omega)\cos(\omega_c t)$ yields the baseband signal and another SSB signal with carrier $2\omega_c$. A low-pass filter will suppress the unwanted SSB terms.

Conclusions

- Hilbert Transform
- Single Side Band (SSB) signals
- Modulation and demodulation of SSB signals