Imperial College London

EE2 Signals and Linear Systems

Pier Luigi Dragotti
Electrical & Electronic Engineering Department
Imperial College London

URL: http://www.commsp.ee.ic.ac.uk/~pld/Teaching/
E-mail: p.dragotti@imperial.ac.uk

Aims and Objectives

- "The concepts of signals and systems arise in a variety of fields and the techniques associated with these notions play a central role in many areas of science and technology such as, for example, communications, aeronautics, bio-engineering, energy, circuit design, ect. Although the physical nature of the signals and systems involved in these various disciplines are different, they all have basic features in common. The aim of the course is to provide the fundamental and universal tools for the analysis of signals, and for the analysis and design of basic systems and this independently of the domain of application."

Aims and Objectives

By the end of the course, you will have understood:

- **Basic signal analysis (mostly continuous-time)**
- **Basic system analysis (also mostly continuous systems)**
- **Time-domain system analysis (including convolution)**
- **Laplace and Fourier Transform**
- **System Analysis in Laplace and Fourier Domains**
- **Filter Design**
- **Sampling Theorem and signal reconstructions**
- **Basics on z-transform**

About the course

- Lectures - 15 hours over 8-9 weeks
- Problem Classes – 7-8 hours over 8-9 weeks
- Assessment – 100% examination in June
- Handouts in the form of pdf slides also available at http://www.commsp.ee.ic.ac.uk/~pld/Teaching/ and on BlackBoard
- Discussion board available on Blackboard
- Text Book:

- ***Recommended Text Book:*** B.P. Lathi, "Linear Systems and Signals", 2^{nd} Ed., Oxford University Press

- ***Also useful:*** A.V. Oppenheim & A.S. Willsky "Signals and Systems", Prentice Hall

Lecture 1
Basics about Signals
(Lathi 1.1-1.5)

Pier Luigi Dragotti
Electrical & Electronic Engineering Department
Imperial College London

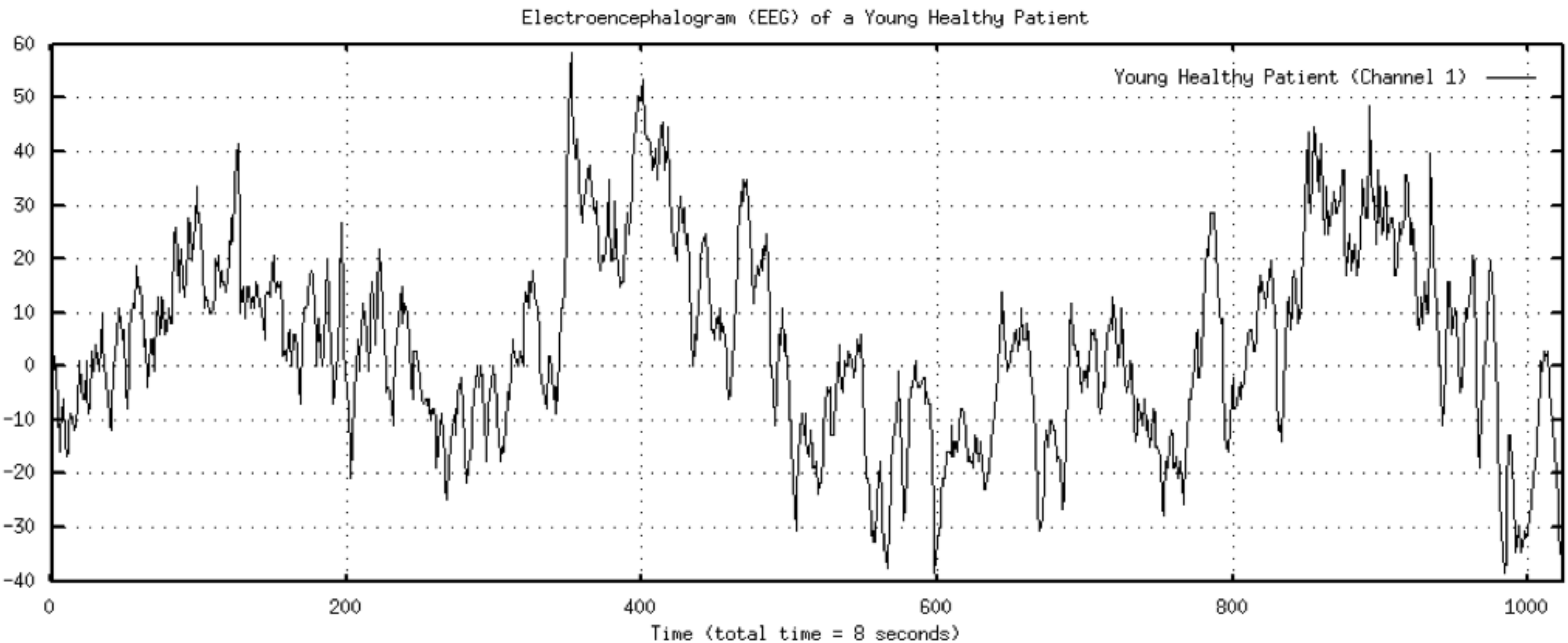
URL: http://www.commsp.ee.ic.ac.uk/~pld/Teaching/
E-mail: p.dragotti@imperial.ac.uk

Outline

- Examples of Signals
- Some useful Signal Operations
- Classification of Signals
- Some useful Signals
  - Unit Step Function
  - Unit Impulse Function
  - The Exponential Function

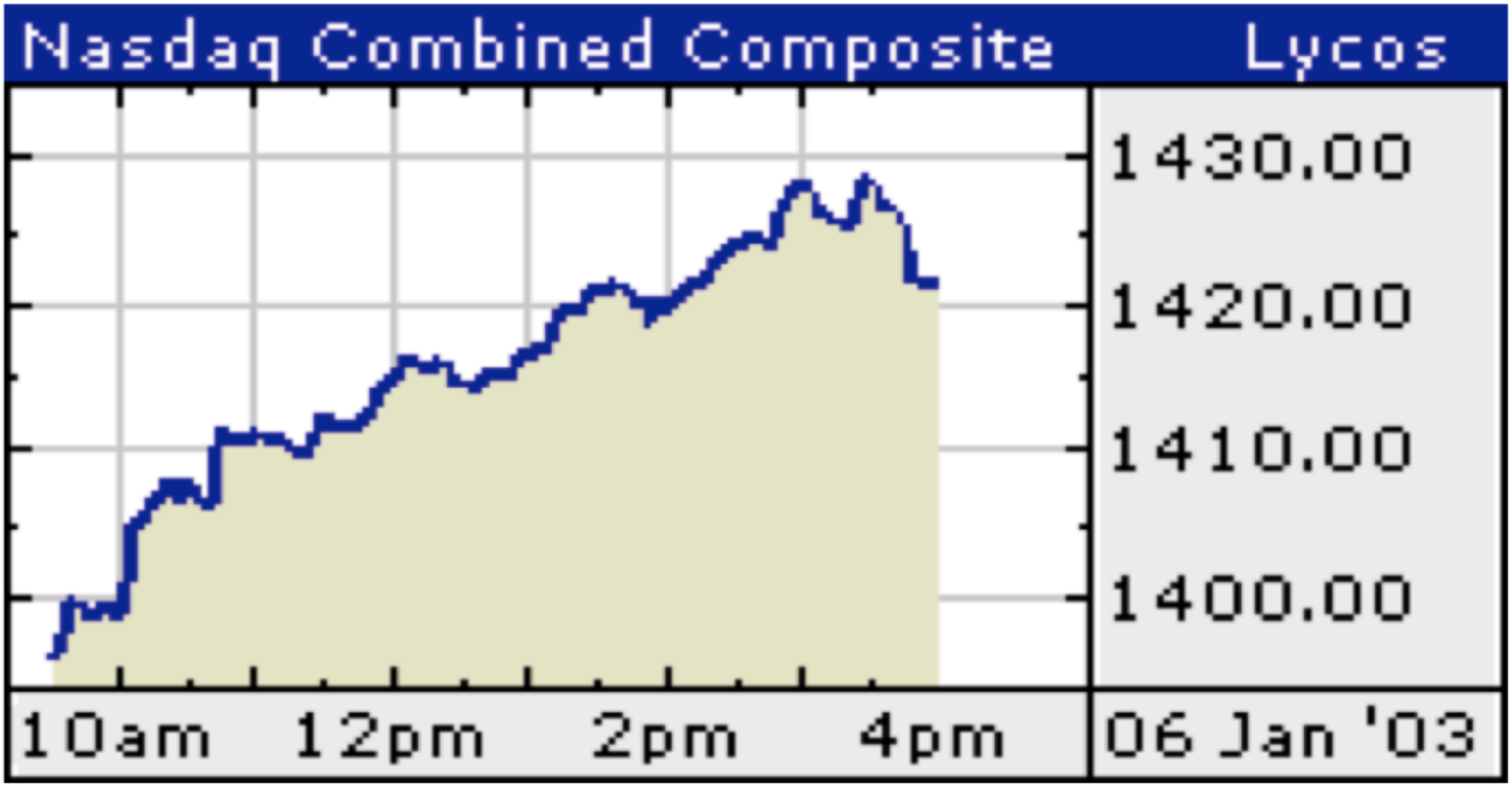
Examples of Signals (1)

- Electroencephalogram (EEG) Signal



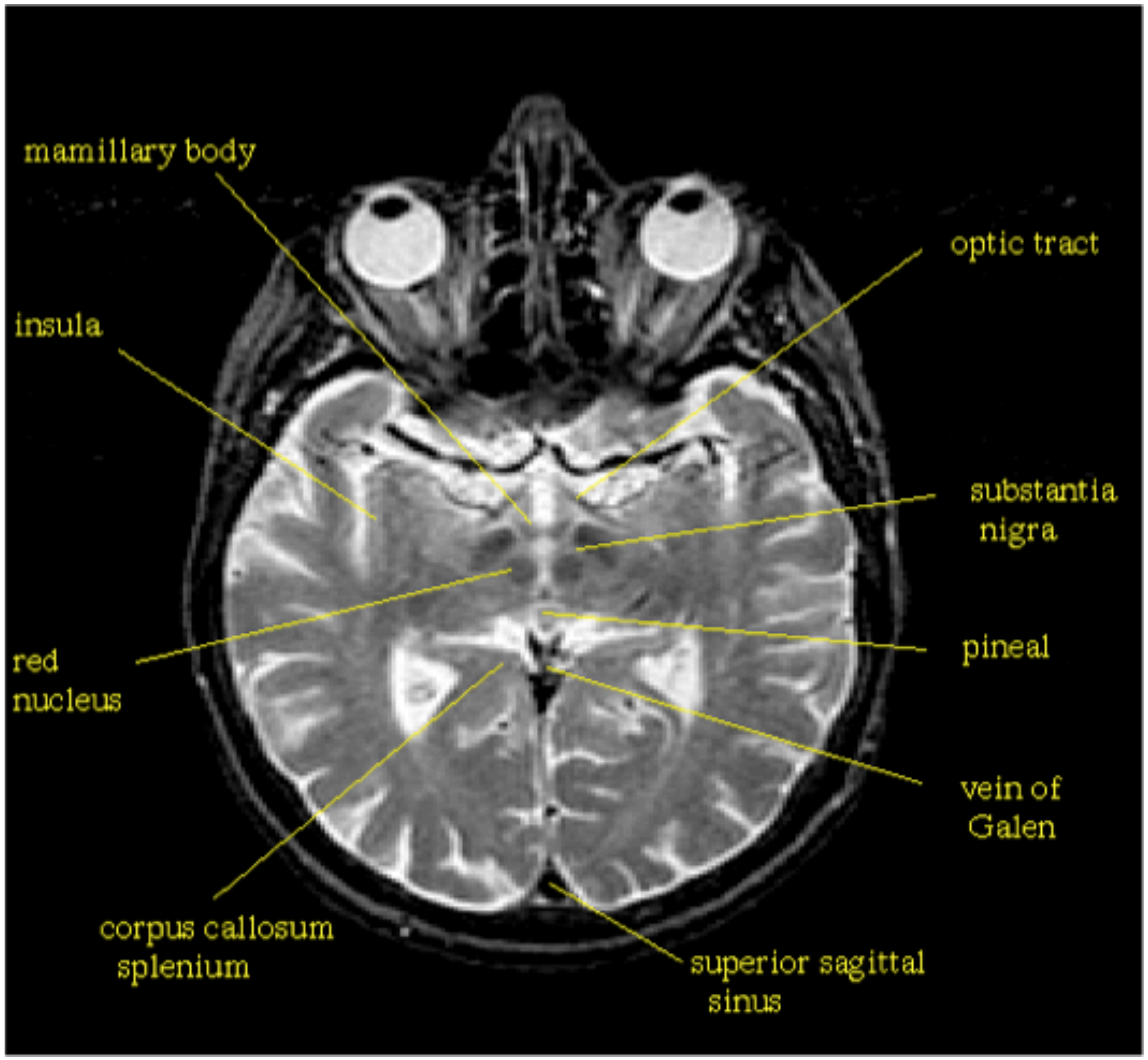
Examples of Signals (2)

- Stock Market Data as a discrete-time signal



Examples of Signals (3)

- Magnetic Resonance Image (MRI) as a 2-dimensional signal



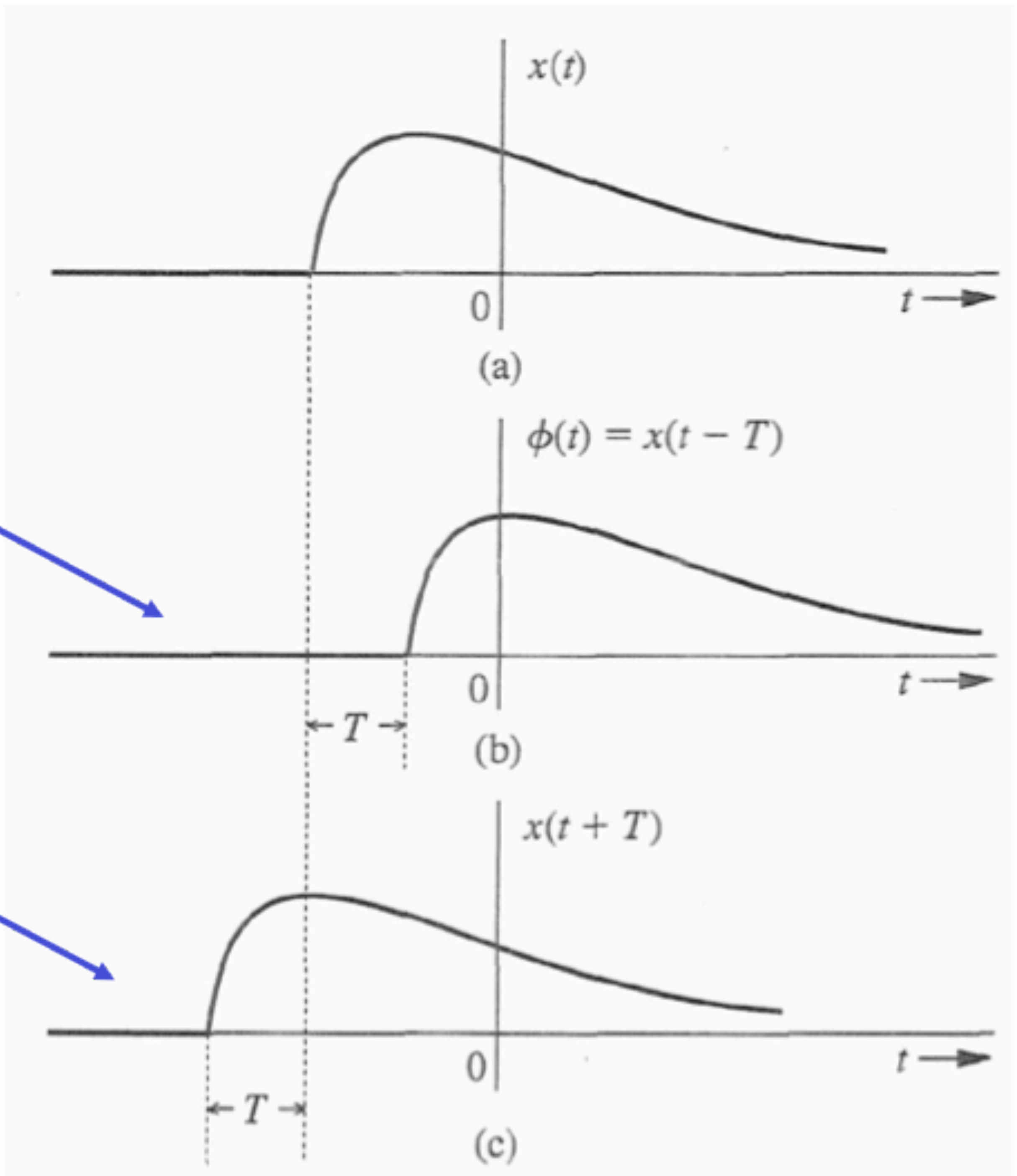
Useful Signal Operations: Time Shifting

- Signal may be delayed by time T:

$$\phi(t+T) = x(t)$$

- or advanced by time T:

$$\phi(t-T) = x(t)$$



L1.2.1

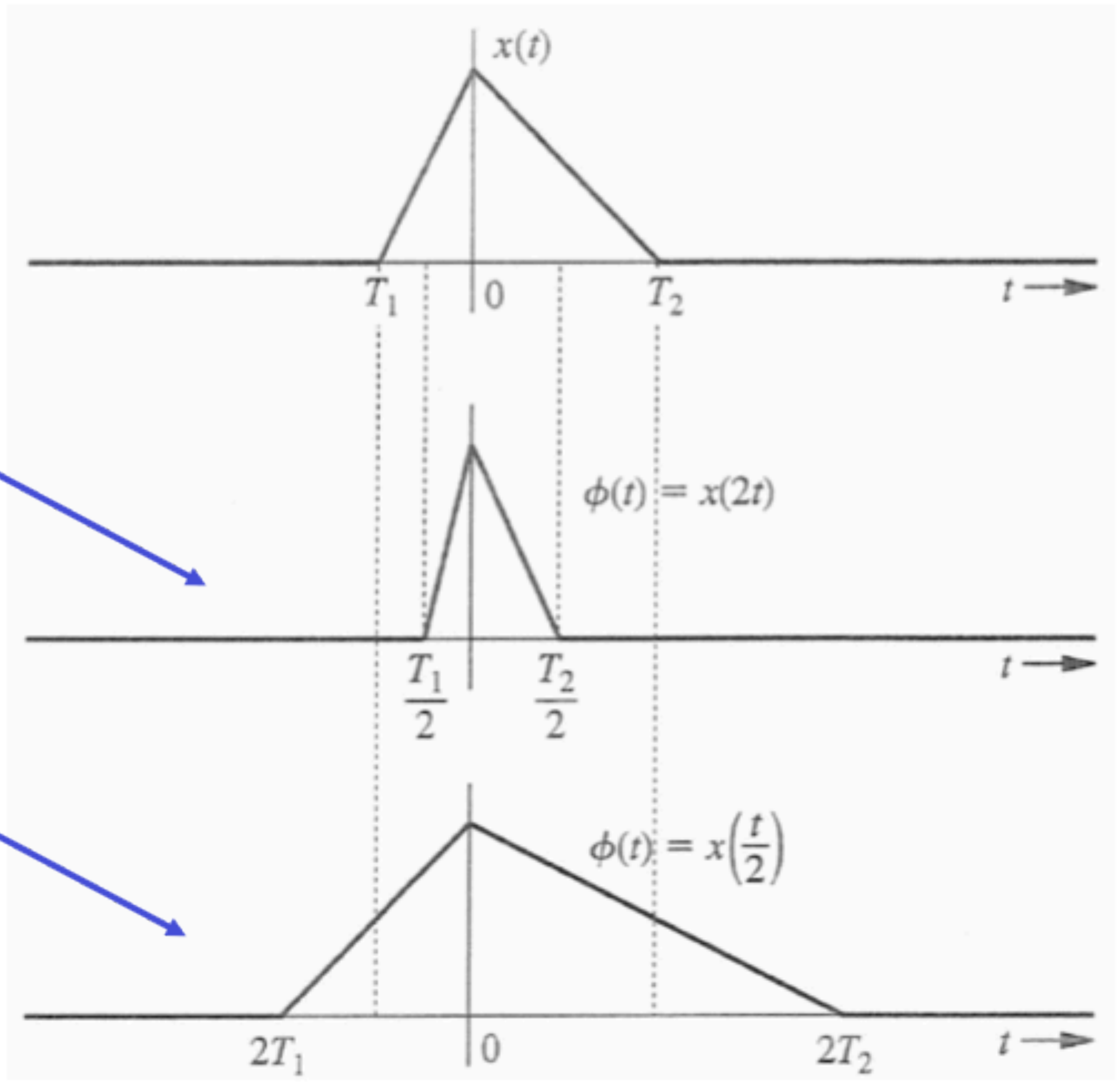
Useful Signal Operations: Time Scaling

- Signal may be compressed in time (by a factor of 2):

$\varphi(t/2)=x(t)$

- or expanded in time (by a factor of 2):

$\varphi(2t)=x(t)$



L1.2.2

Signals Classification

- Signals may be classified into:
  - Energy and power signals
  - Discrete-time and continuous-time signals
  - Analogue and digital signals
  - Deterministic and probabilistic signals
  - Periodic and aperiodic signals
  - Even and odd signals

Signals Classification: Energy vs Power

Size of a signal *x(t)*

- Measured by signal energy E_x:

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$$

- Generalize for a complex valued signal to:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

- Energy must be finite, which means

$$\text{signal amplitude} \to 0 \text{ as } |t| \to \infty$$

Lathi Section 1.1

L1.1

Signals Classification: Energy vs Power (2)

- If amplitude of *x(t)* does not → 0 when t → ∞, need to measure power *Px* instead:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

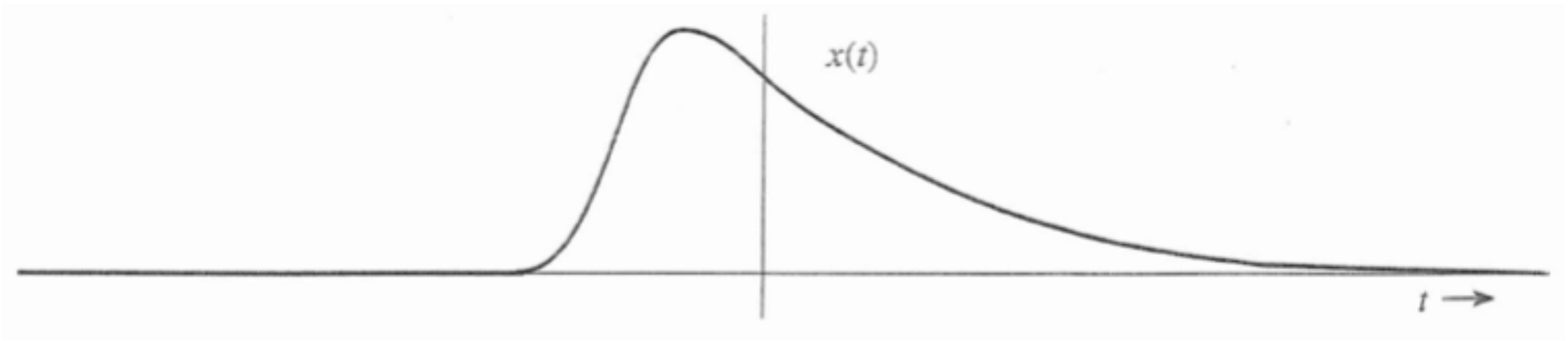
- Again, generalize for a complex valued signal to:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

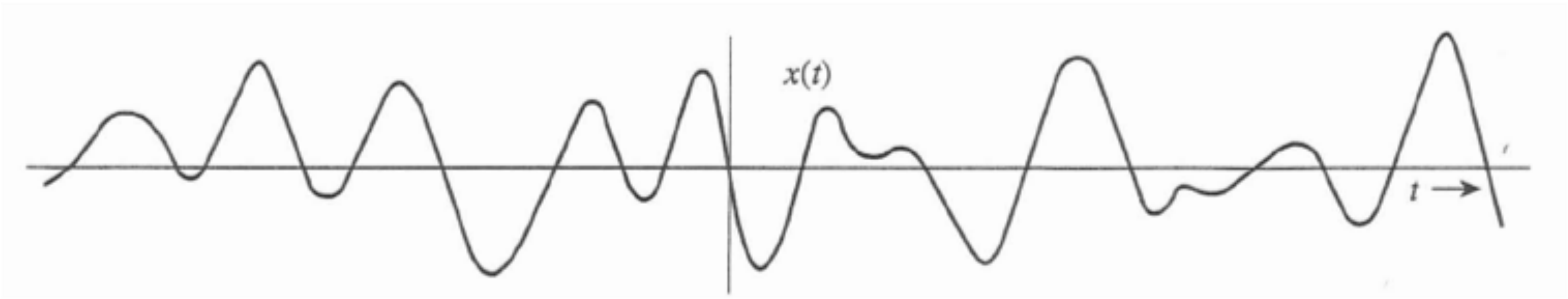
L1.1

Signals Classification: Energy vs Power (3)

- Signal with finite energy (zero power)



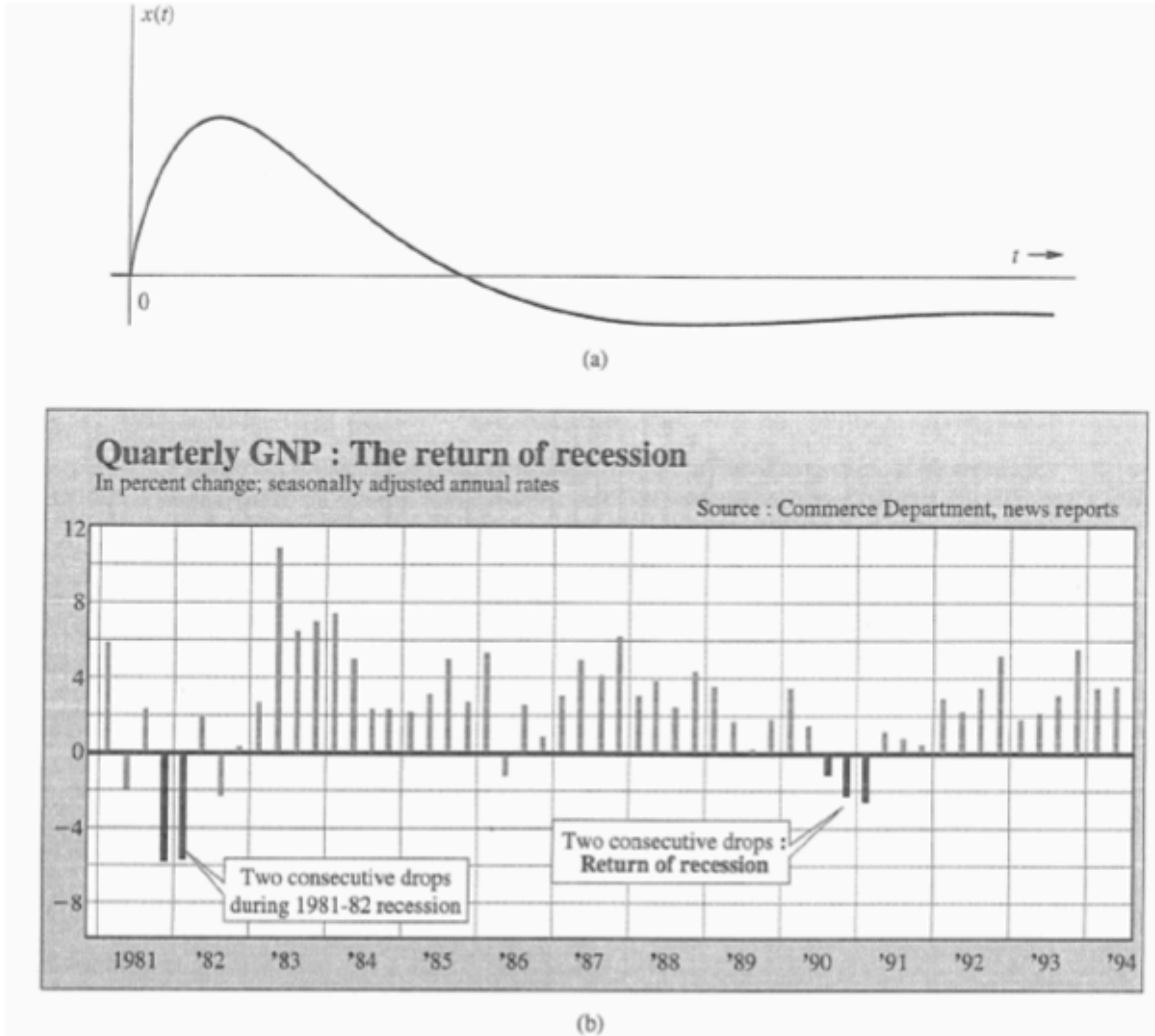
- Signal with finite power (infinite energy)



L1.1

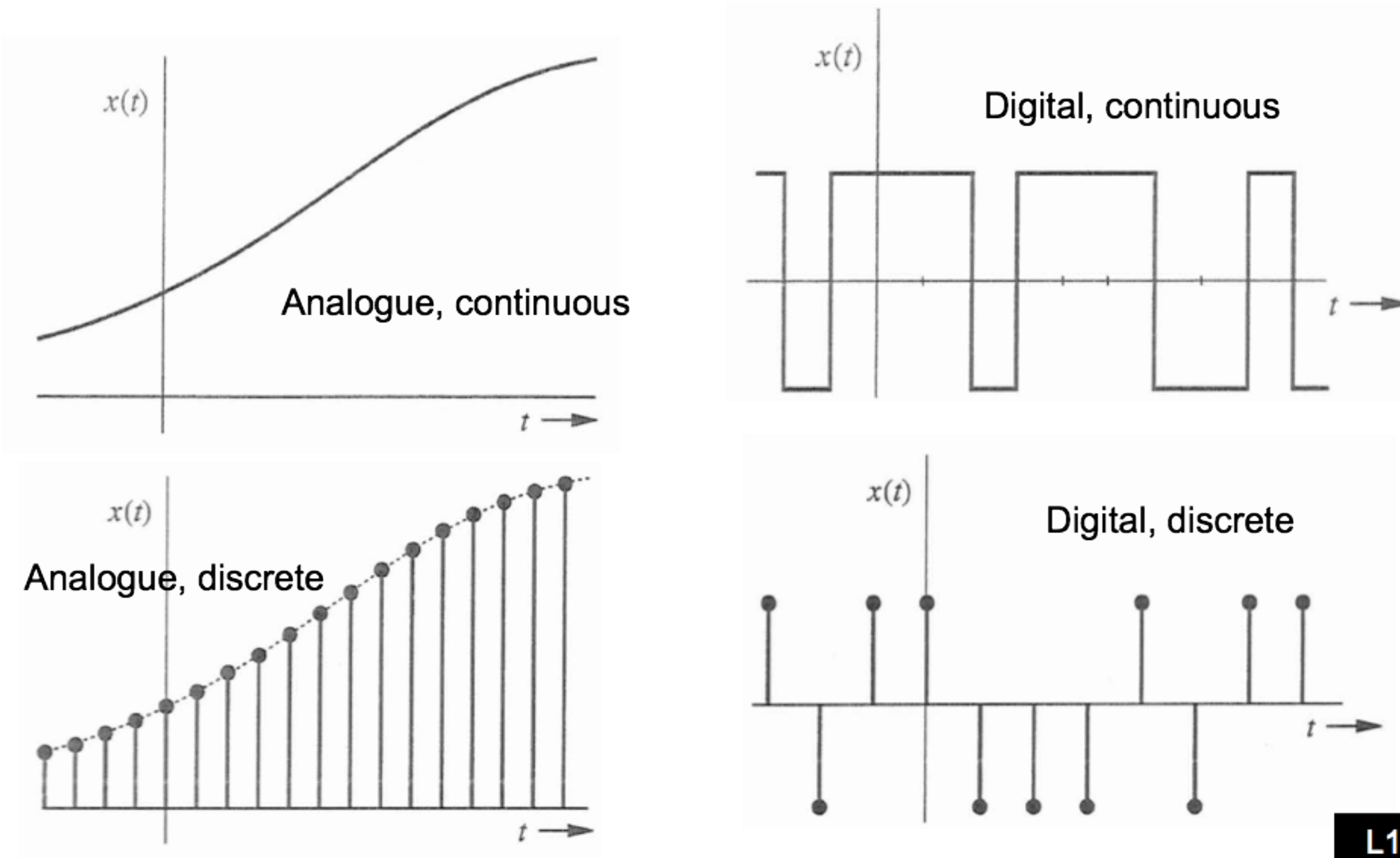
Signals Classification: Continuous-time vs Discrete-time

- Continuous-time
- Discrete-time



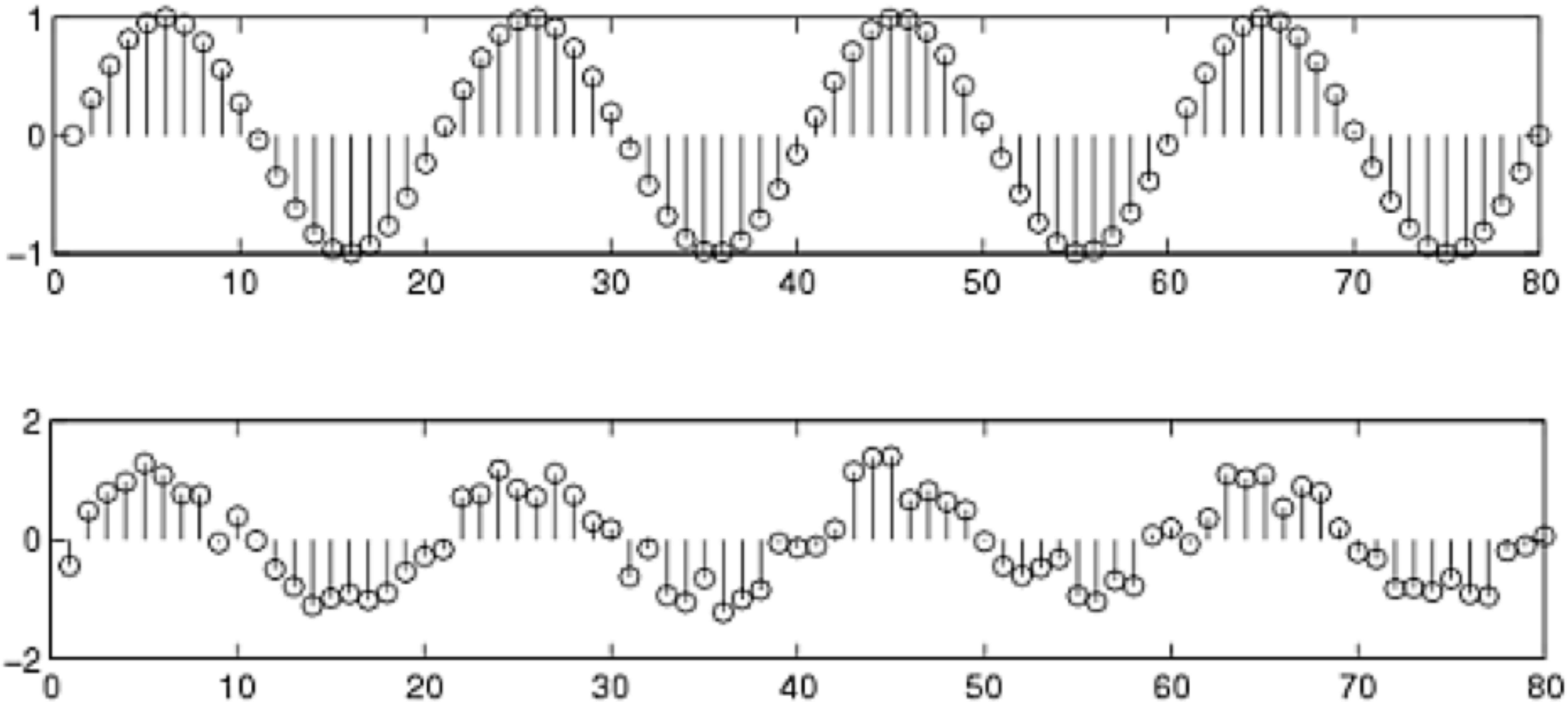
L1.3

Signals Classification: Analogue vs Digital



L1.3

Signals Classification: Deterministic vs Random



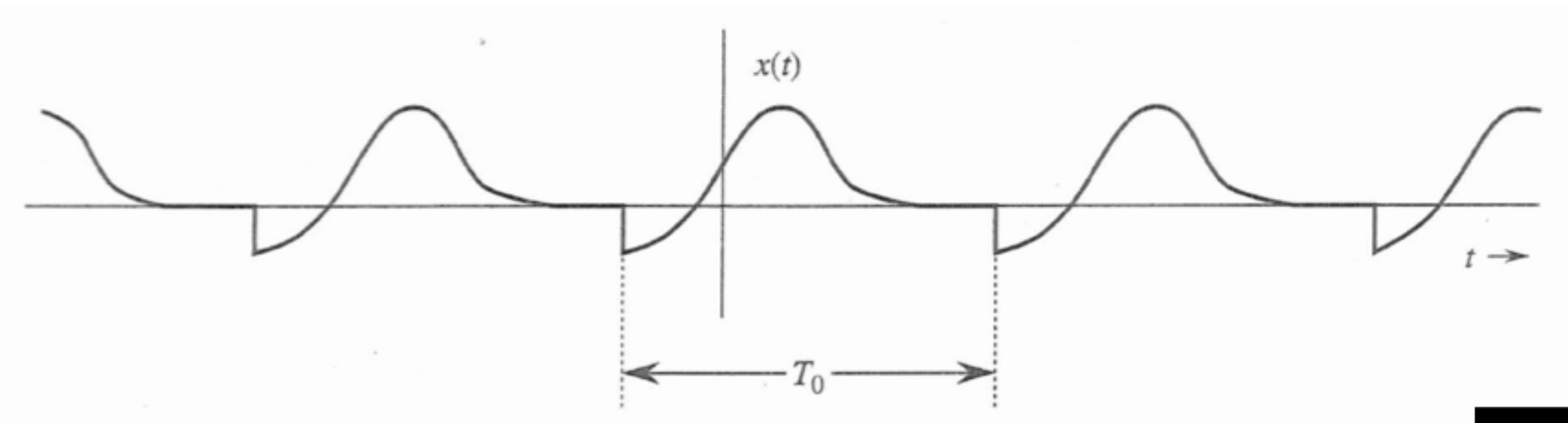
L1.3

Signals Classification: Periodic vs Aperiodic

- A signal *x(t)* is said to be periodic if for some positive constant T_o

$$x(t) = x(t + T_0) \qquad \text{for all } t$$

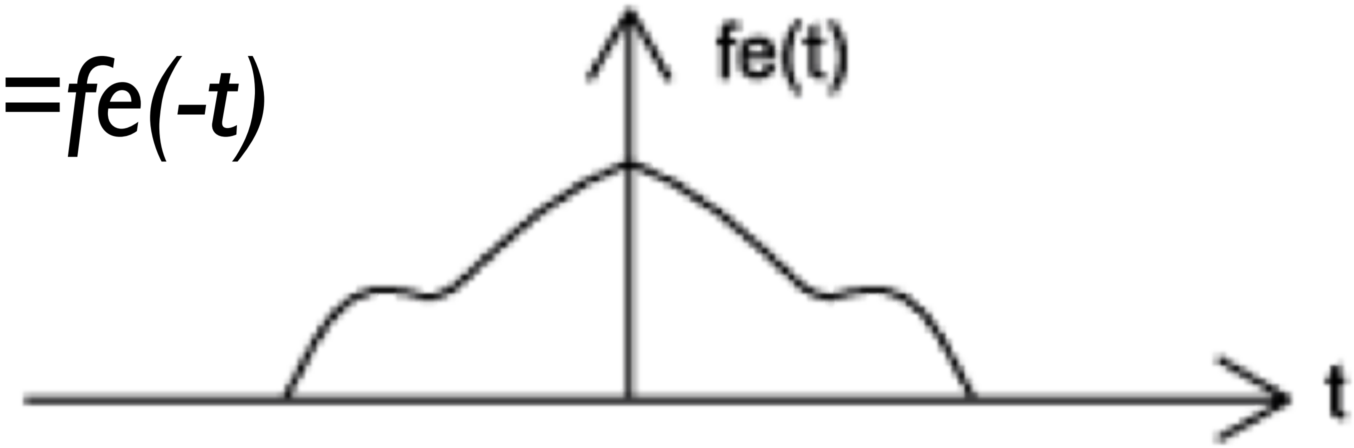
- The smallest value of T_o that satisfies the periodicity condition of this equation is the *fundamental period* of x(t).



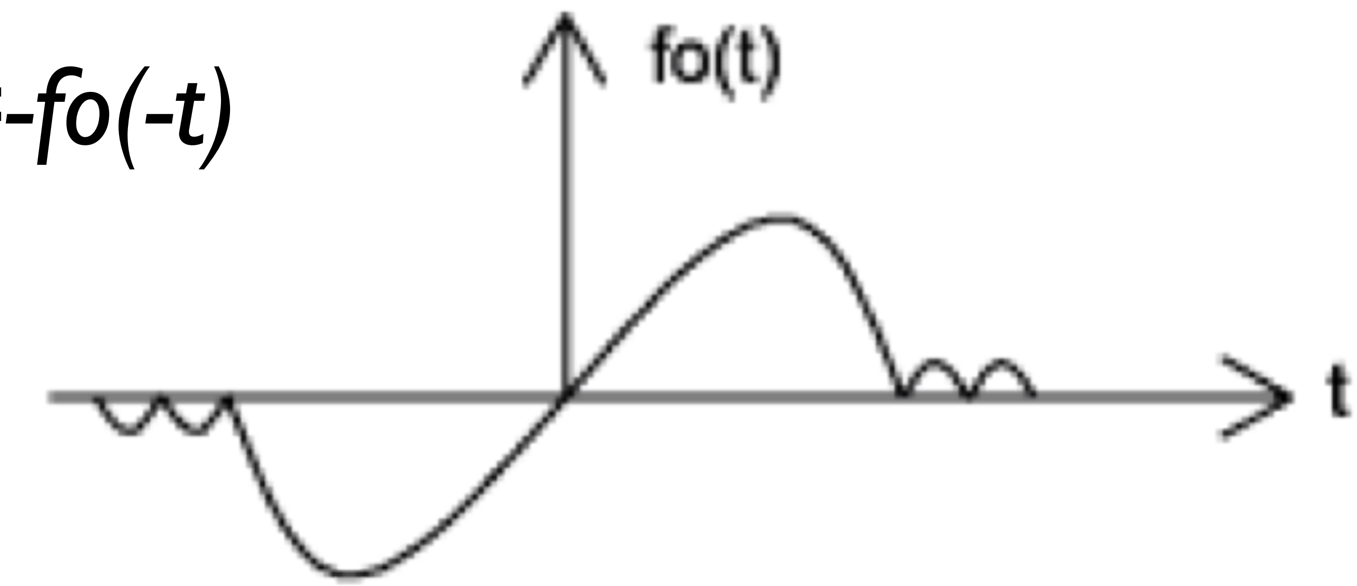
L1.3

Signals Classification: Even vs Odd

$fe(t)=fe(-t)$



$fo(t)=-fo(-t)$



Signals Classification: Even vs Odd (2)

- Even and odd functions have the following properties:
  - Even x Odd = Odd
  - Odd x Odd = Even
  - Even x Even = Even

- Every signal *x(t)* can be expressed as a sum of even and odd components because:

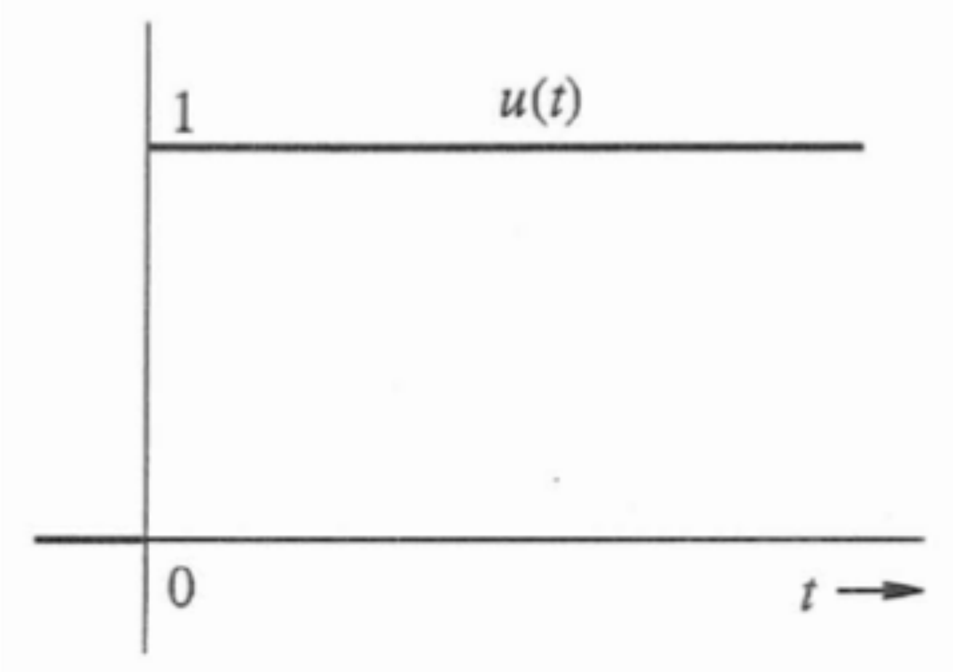
$$x(t) = \underbrace{\tfrac{1}{2}[x(t) + x(-t)]}_{\text{even}} + \underbrace{\tfrac{1}{2}[x(t) - x(-t)]}_{\text{odd}}$$

L1.5

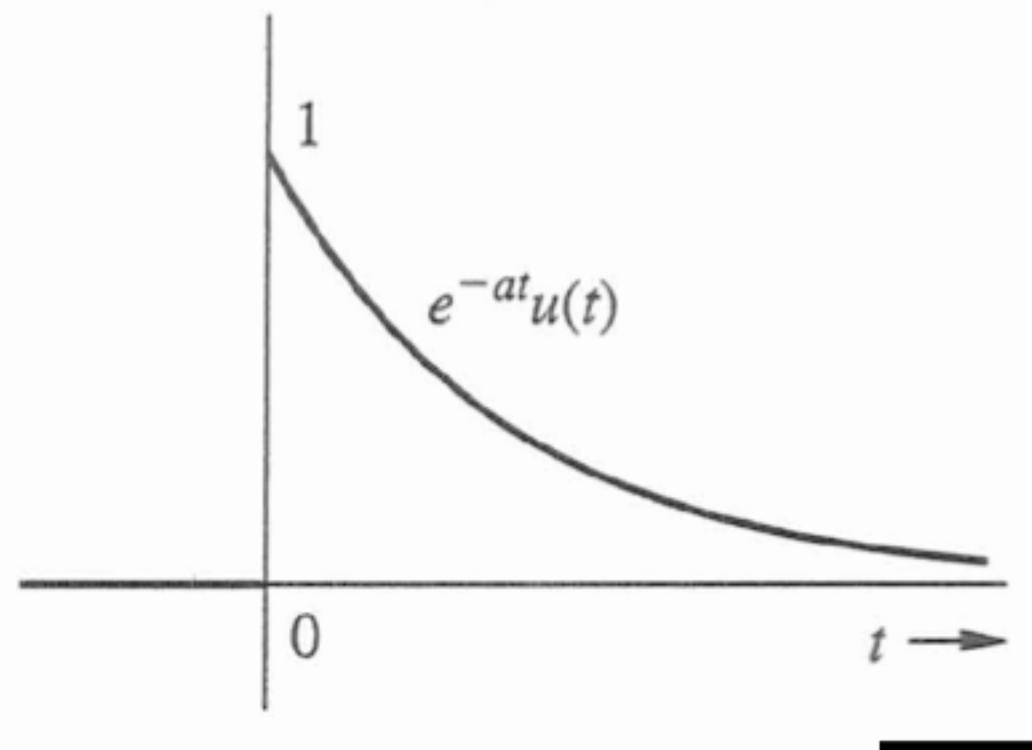
Useful Signals: Unit Step Function *u(t)*

- Step function defined by:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



- Useful to describe a signal that begins at t = 0 (i.e. causal signal).
- For example, the signal represents an everlasting exponential that starts at t = -∞.
- The causal for of this exponential can be described as:

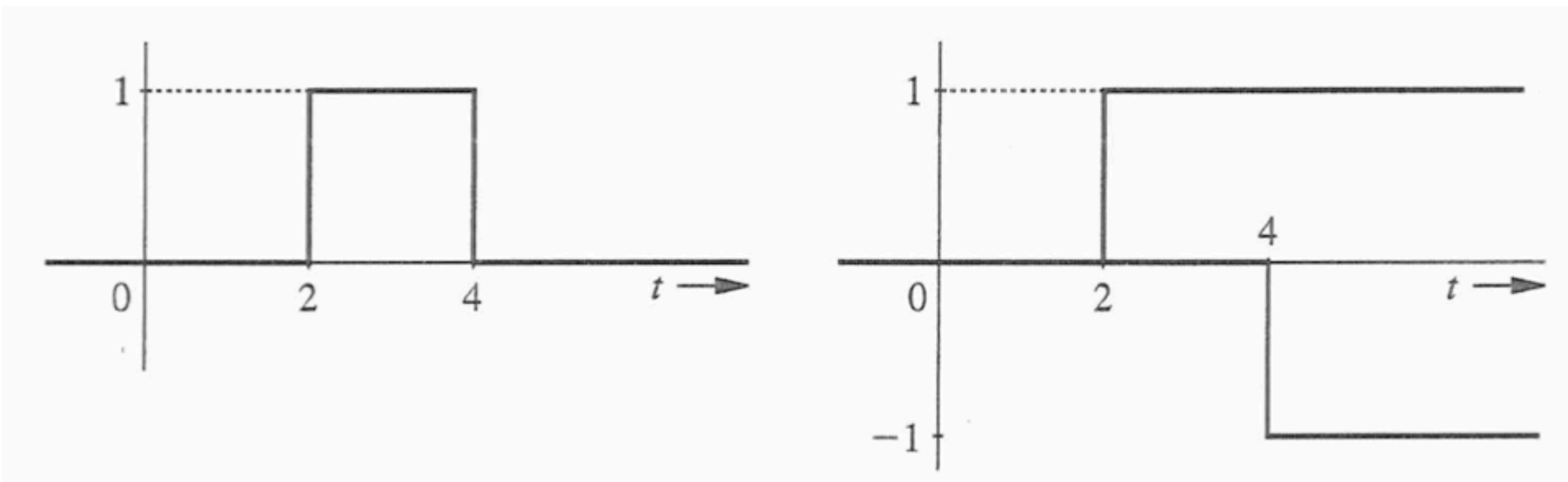


L1.4.1

Useful Signals: Pulse Signal

A pulse signal can be represented using two step functions. E.g:

$$x(t) = u(t-2) - u(t-4)$$



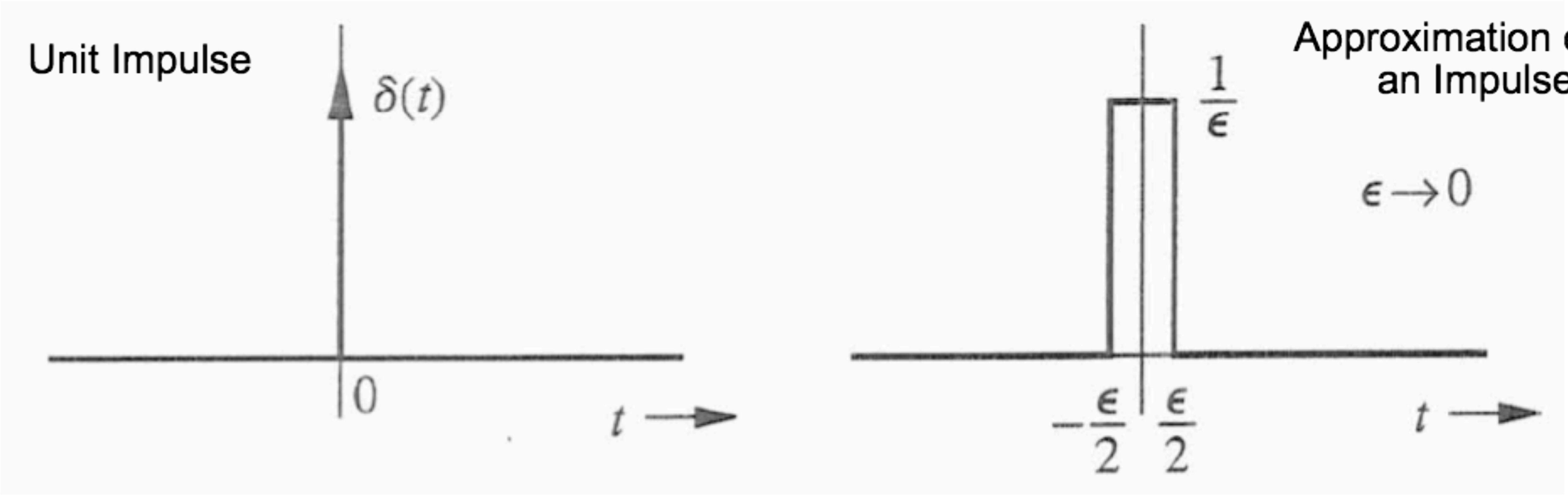
L1.4.1

Useful Signals: Unit Impulse Function *δ(t)*

- First defined by Dirac as:

$$\delta(t) = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



L1.4.2

Sampling Property of the Unit Impulse Function

- Since impulse is non-zero only at t = 0, and Φ(t) at t = 0 is Φ(0), we get:

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$

It follows that:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)\,dt = \phi(0)\int_{-\infty}^{\infty} \delta(t)\,dt$$
$$= \phi(0)$$

If we want to sample *φ(t)* and *t=T*, we multiply by *δ(t-T):*

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)\,dt = \phi(T)$$

L1.4.2

The Exponential Function e^{st} (1)

- The exponential function is very important in signals & systems.
- The parameter s is a complex variable given by:

$$s = \sigma + j\omega$$

Therefore

$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t} = e^{\sigma t}(\cos \omega t + j \sin \omega t)$$

Since $s^* = \sigma - j\omega$ (the conjugate of s), then

$$e^{s^* t} = e^{\sigma - j\omega} = e^{\sigma t} e^{-j\omega t} = e^{\sigma t}(\cos \omega t - j \sin \omega t)$$

and

$$e^{\sigma t} \cos \omega t = \frac{1}{2}(e^{st} + e^{s^* t})$$

L1.4.3

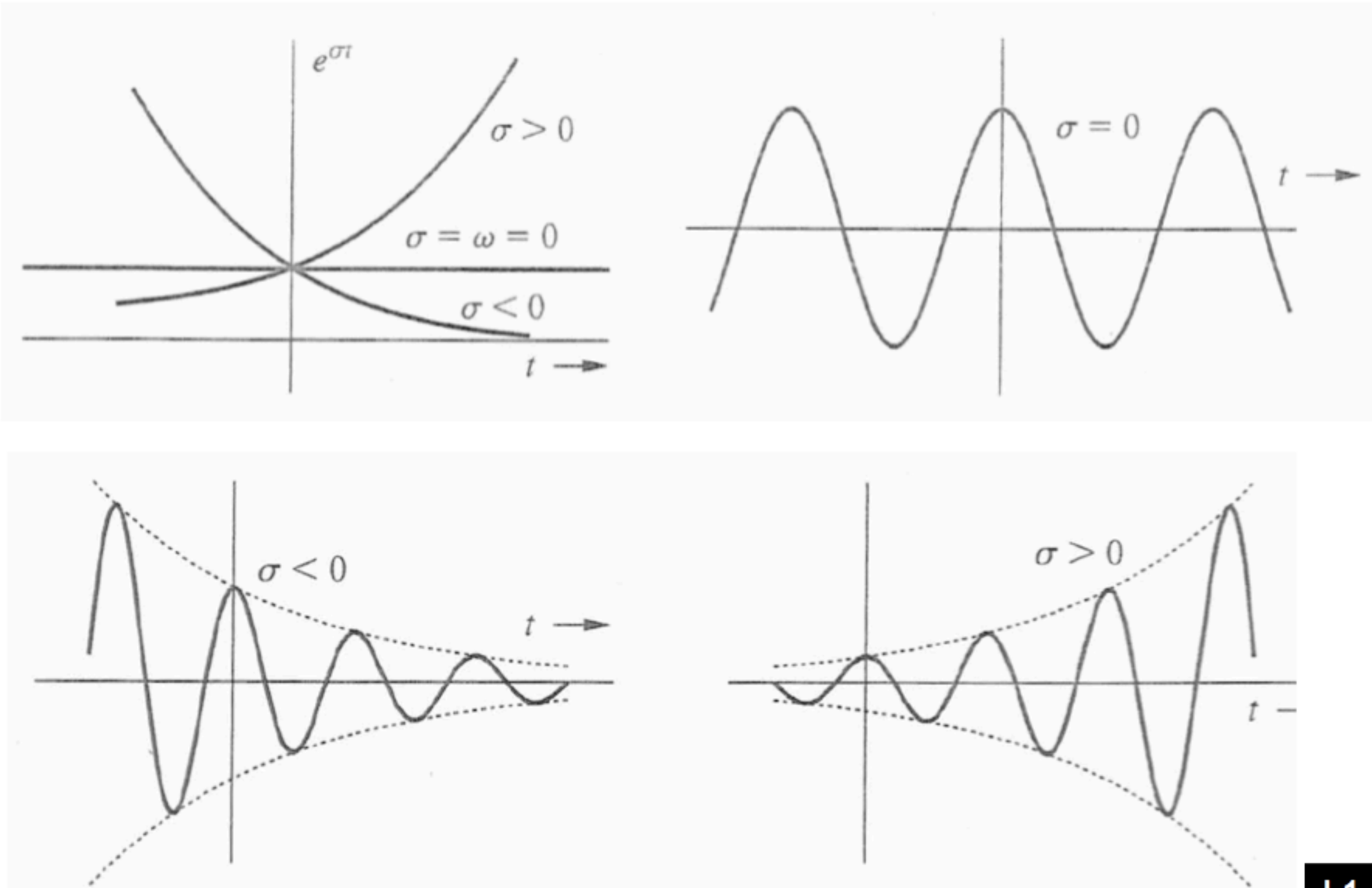
The Exponential Function e^{st} (2)

- The exponential function e^{st} can be used to model a large class of signals.
- Here are a few examples:

1. A constant $k = ke^{0t}$ $\quad (s = 0)$
2. A monotonic exponential $e^{\sigma t}$ $\quad (\omega = 0,\ s = \sigma)$
3. A sinusoid $\cos \omega t$ $\quad (\sigma = 0,\ s = \pm j\omega)$
4. An exponentially varying sinusoid $e^{\sigma t} \cos \omega t$ $\quad (s = \sigma \pm j\omega)$

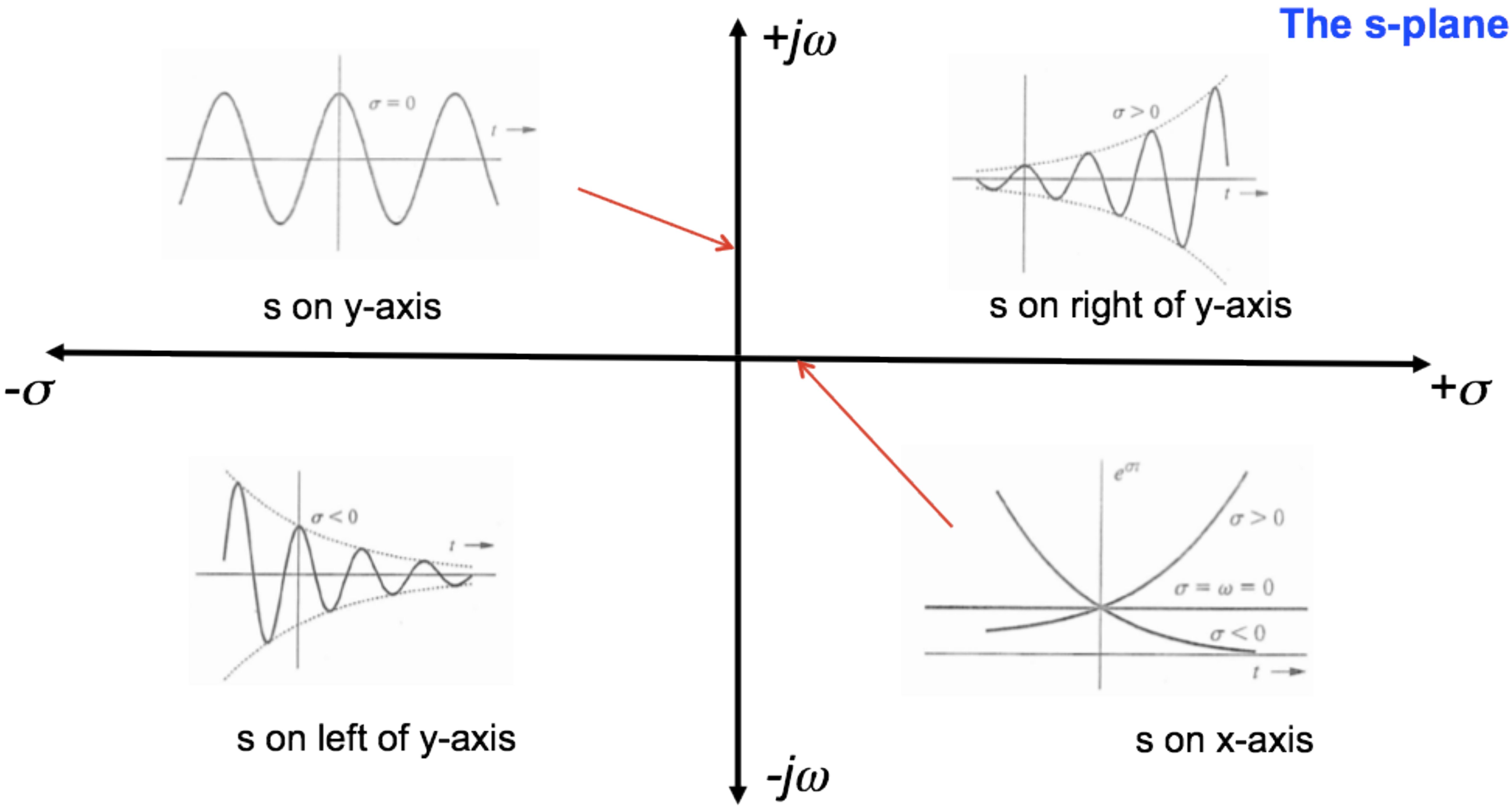
L1.4.3

The Exponential Function e^{st} (3)



L1.4.3

The Complex Frequency Plane $s=\sigma+j\omega$



L1.4.3

Discrete-Time Exponential γ^n

- A continuous-time exponential e^{st} can be expressed in alternate form as

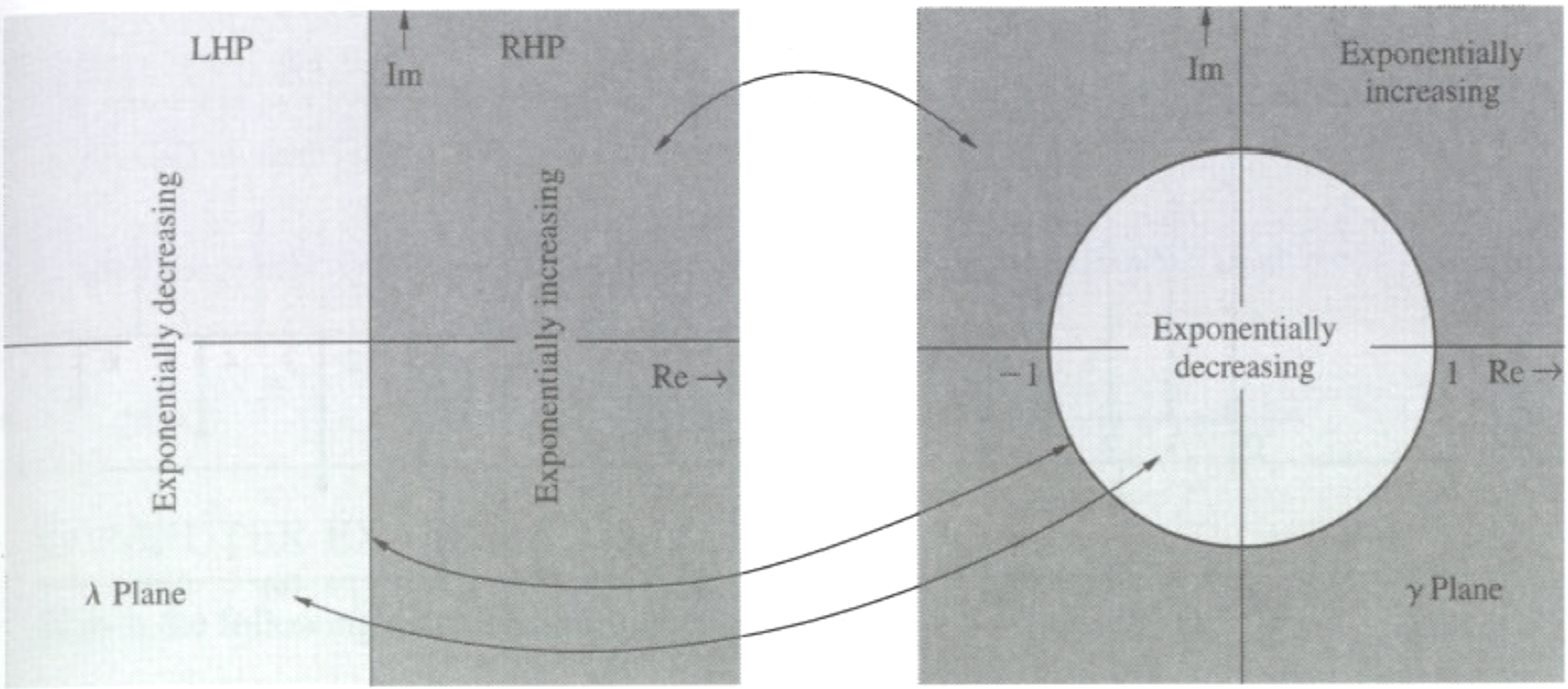
$$e^{st} = \gamma^t$$

with $\gamma = e^s$

- Similarly we have for discrete-time exponentials $e^{\lambda n} = \gamma^n$
- The form γ^n *is preferred for discrete-time exponentials*
- *When Re{λ}<0, then* $|\gamma| < 1$ and the exponential decays
- When *Re{λ}>0, then* $|\gamma| > 1$ and the exponential grows
- When λ is purely imaginary then $|\gamma| = 1$ and the exponential is constant-amplitude and oscillates

L3.3-3 p254

Discrete-Time Exponential γ^n (cont'd)



(a)

(b)

L3.3-3 p255