International Journal of Neural Systems, Vol. 18, No. 2 (2008) 1–8

ONLINE DETECTION OF THE MODALITY OF COMPLEX-VALUED REAL WORLD SIGNALS

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A novel method for the online detection of the modality of complex-valued nonlinear and nonstationary signals is introduced. This is achieved using a convex combination of complex nonlinear adaptive filters with different transient characteristics. To facilitate the online mode of operation, the convex mixing parameter λ within the proposed architecture is made gradient adaptive. Our focus is on the most important aspect of complex nonlinear modeling, that is, the identification of the split-complex and fully-complex nature of the signal in hand. The algorithms derived are robust and capable of tracking the changes in the modality of both benchmark and real world radar and wind complex vector fields.

Keywords:

1. Introduction

Statistical methods for the characterisation of signal modality have been introduced in Physics some ten years ago[1] and are gradually being adopted in neuroscience and engineering.[2,3] This has been achieved mainly for univariate real-valued signals, with some recent extensions to multivariate real-valued signals.[1] This way, we can establish a rigorous statistical framework for the assessment of signal modality, that is, their linear or nonlinear nature, together with their deterministic and stochastic nature. These, hypothesis testing based, methods have proven extremely useful in practical applications,[4,5] but despite their computational power their applications are limited to offline processing and stationary signals.

On the other hand, recent developments in multidimensional signal processing have highlighted the need for novel and nonlinear frameworks for the processing of complex-valued signals.[6,7] Statistical tests for complex signal modality characterisation in an offline manner already exist[8] and have found their application in biomedicine and engineering. In that context, it has been shown that the knowledge of changes in the signal nature between e.g. linear and nonlinear, deterministic and stochastic and split- and fully-complex, are extremely useful for the choice of the actual online processing architectures. It was also realised that the development of methods for the online tracking of those changes would have a huge potential in a plethora of applications including medical, seismic, sonar and radar. At present, however, there are no available online signal modality characterisation methods for complex-valued signals.

As a first step in this direction, we set out to develop a new scheme for the online detection of the changes in the type of nonlinearity of complex-valued signals. This is achieved as an extension of our earlier results on the online tracking of the nature of real-valued signals.[9] It is intuitively clear that in

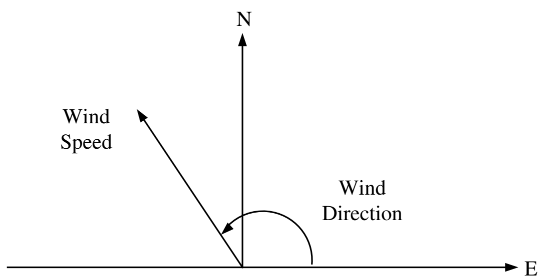


Fig. 1. Wind recordings: A complex valued representation.

order to discriminate between the split- versus fully-complex nature of a nonlinear complex time series, we can present the signal to two adaptive filters operating in parallel, and possessing respectively a split- and a fully-complex activation function (AF).[10] The comparison of the mean square error at the output of the two filters can then be used as a means of identifying the type of nonlinearity present in the complex-valued signal. Whilst this solution caters for a simple way of determining the type of nonlinearity, there are ambiguities due to the need to choose many parameters of the corresponding filters. Our approach resolves this issue of high parametric dependence by relying on the "synergy" between the two filters and is based on the gradient descent methodology. Our analysis is supported by simulations on real world complex-valued radar and wind signals.

2. Open Problems in the Processing of Real World Complex Data

To illustrate the need for the development of new algorithms for complex-valued signal modeling, consider the wind signal, which is typically modeled as a bivariate process of its speed and direction.[11] It is clear from Fig. 1, that a wind measurement can be represented as a vector of its speed $v(k)$ and direction $d(k)$ components, in the $N \div E$ (North/East) coordinate system. There is a clear interdependence between wind signal components, a fact that is not taken into account in the current approaches to wind forecasting.

Although both the speed and direction are integral components of the wind signal, in practical applications, only the speed component is taken into account, hence introducing a systematic error in forecasts.[11] This suggests the use of multidimensional networks developed in an associative and division algebra, in which the universal function approximation theorems do exist. This is exactly the case with complex algebra. Indeed, from Fig. 1, the wind vector $\mathbf{v}(k)$ can be expressed in the complex domain $\mathbb{C}$ as

$$\mathbf{v}(k) = |\mathbf{v}(k)|\, e^{\jmath d(k)} = v_E(k) + \jmath v_N(k) \qquad (1)$$

To clarify this, observe that the two wind components speed v and direction d (which are of different natures) are modeled as a single quantity in a complex representation space $\mathbb{C}$. It is also clear that for intermittent processes, the nature of the measurements changes according to the signal (wind) regime (e.g. breeze, gust). Methods for testing the nature of such signals are in their infancy; statistical tests for the variable dependence within a signal model (bivariate vs. complex valued) can be found in Ref. 8.

2.1. *Split-complex versus fully-complex activation functions*

Figure 2 shows the block diagram of a nonlinear finite impulse response (FIR) adaptive filter. A critical issue in the applicability of the nonlinear gradient descent algorithm in the complex domain is that of the choice of the activation function Φ. The main challenge there has been the lack of bounded and analytic complex nonlinear activation functions in $\mathbb{C}$. Initial results have employed non-analytic but bounded nonlinear activation functions, typically adopting a split-complex function, whereby the real and imaginary component of the net input $net(k) = \mathbf{x}^T(k)\mathbf{w}(k)$ are separated and fed through

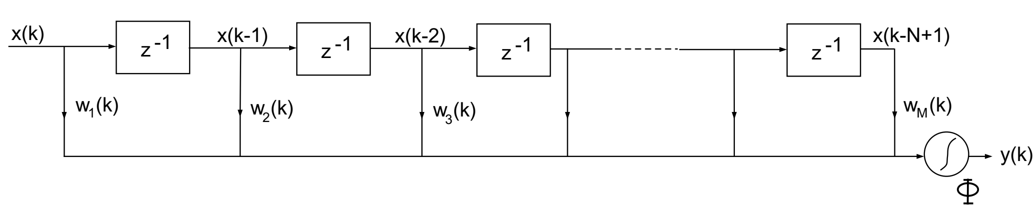


Fig. 2. The nonlinear finite impulse response (FIR) adaptive filter.

the dual real valued activation function $f_R(x) = f_I(x)$, $x \in \mathbb{R}$. A split-complex AF can therefore be represented as

$$\Phi_{split}(z) = f_R(z^r) + jf_I(z^i) = u(z^r) + jv(z^i), \quad (2)$$

where $(\cdot)^r$ and $(\cdot)^i$ denote respectively the real and imaginary components of a complex quantity. From (2), it can be seen that the real and imaginary components at the output of the nonlinearity are functions of only the real and the imaginary part of z respectively. This indicates that, for instance, we cannot use the Cauchy-Riemann equations to calculate the gradients within the weight updates of such filters (for a full derivation of split-complex learning rules, see e.g. Ref. 12).

A fully-complex activation function is basically a standard complex function of complex variables and contrary to the split-complex nonlinearity, it is well suited to the modeling of signals with high component correlations (for more detail, refer to Ref. 13).

2.2. *Nonlinearity within real world complex signals*

Depending on the level of correlation between the components of real world complex-valued signals, we can differentiate between the following approaches to the modeling of such signals:

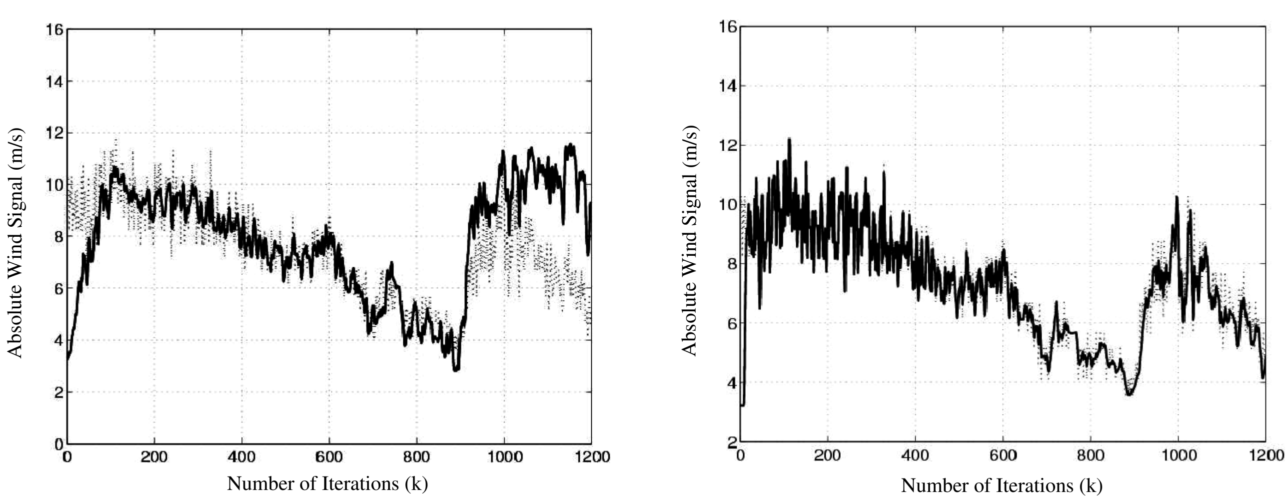
(i) The standard *dual univariate* approach, where the wind signal components (speed and direction) are predicted separately and then put back together to form a complex vector;
(ii) The *split*-complex approach, where the wind speed and direction are modeled in the complex algebra, with the split-complex nonlinear activation of neurons;
(iii) The *fully*-complex representation in the complex algebra, with a general complex-valued nonlinear activation function of neurons.

We next illustrate the importance of testing for component dependencies within the proposed framework, by comparing the one-step ahead forecasting performances using two neural networks with respectively the split- and fully-complex activation function. The results of the simulations are shown in Fig. 3 and indicate the predominantly fully-complex nature of the wind signal.[10,14]

Although the above simulations were conducted in an online fashion, the split-complex and fully-complex based predictors did not collaborate. We therefore need to extend these results and design an architecture in which the actual subfilters would operate in a collaborative manner in order to indicate the changes in the signal nature online.

3. Complex Hybrid Filters

Following the results from Refs. 9, 15 and 16, we propose to use a combination of two adaptive filters trained respectively by the split- and fully-complex



(a) (b)

Fig. 3. Neural network based prediction; (a) *Split*-complex approach; (b) *Fully*-complex approach; Solid line: predicted signal; Broken line: original signal.

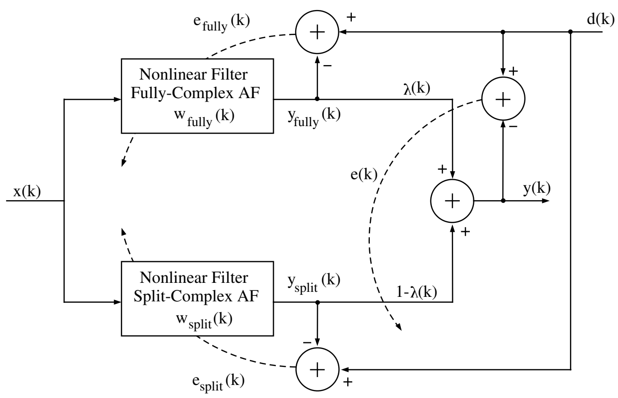


Fig. 4. Convex combination of a fully-complex and split-complex nonlinear gradient descent algorithm. Mixing parameter $\lambda(k)$, $0 \leq \lambda(k) \leq 1$ ensures the overall output $|y(k)| \in [|y_{fully}(k)|, |y_{split}(k)|]$.

nonlinear gradient descent algorithm. For simplicity, we consider feedforward structures, as shown in Fig. 4. Whereas previous hybrid filtering approaches focused on improving the performance of such a filter as compared with that of constitutive subfilters,[16] our approach is radically different[9] in the sense that we are only interested in the behaviour of the mixing parameter $\lambda(k)$ and the only requirement on the constitutive subfilters is for them not to diverge. The two filters are trained online, using their respective instantaneous output errors. Unlike the result from Sec. 2.2, the two filters collaborate and operate in parallel, which permits tracking of the split-/fully-complex nature of a nonlinear complex signal. The mixing parameter λ from Fig. 4 takes values in the range of $[0, 1]$ hence preserving convexity.[a]

3.1. *Learning algorithms for the subfilters*

Training of the two nonlinear finite impulse response (dynamical perceptron) subfilters from Fig. 4 is performed using the Nonlinear Grading Descent (NGD) algorithm. The NGD can be described by

$$\begin{aligned} e(k) &= d(k) - y(k) \\ y(k) &= \Phi(net(k)) \\ net(k) &= \mathbf{x}^T(k)\mathbf{w}(k) \\ \mathbf{w}(k+1) &= \mathbf{w}(k) - \eta\nabla_{\mathbf{w}}E(k) \end{aligned} \tag{3}$$

where $e(k)$ is the instantaneous error at the output of the filter for the time instant k, $d(k)$ is the desired signal, $y(k)$ is the output, $\mathbf{x}(k) = [x(k-1), \ldots, x(k-N)]^T$ is the input signal vector, $\mathbf{w}(k) = [w_1(k), \ldots, w_N(k)]^T$ is the filter coefficient (weight) vector, N is the length of the filter and $(\cdot)^T$ denotes the vector transpose operator. $e(k)$, $d(k)$, $\mathbf{x}(k)$, and $\mathbf{w}(k)$ are all complex-valued. $\Phi(\cdot)$ denotes the complex nonlinear activation function, the (real) parameter η denotes the learning rate (step-size) critical to the convergence and dynamical behaviour of the NGD. $E(k)$ is the cost function given by

$$E(k) = \frac{1}{2}|e(k)|^2. \tag{4}$$

Following the standard complex Least Mean Square (LMS) derivation,[17] for a general (fully-complex) nonlinear activation function, the weight update can be expressed as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \eta e(k)(\Phi'[net(k)])^*\mathbf{x}^*(k). \tag{5}$$

where $(\cdot)'$ and $(\cdot)^*$ respectively denote the complex differentiation and complex conjugation operators. The upper and lower subfilters from Fig. 4 are updated using the NGD algorithm with a fully- and split-complex activation function respectively.

3.2. *Learning algorithm for the complex hybrid filter*

From Fig. 4, the output of the hybrid filter can be expressed as

$$y(k) = \lambda(k)y_{fully}(k) + (1-\lambda(k))y_{split}(k) \tag{6}$$

where $y_{fully}(k)$ and $y_{split}(k)$ are as defined above. The value of the mixing parameter $\lambda(k)$ within each of these structures *adapts* according to the dynamics and nature of the input signal. For instance, the values of λ approaching unity will indicate the fully-complex nature of the signal in hand, whereas the values of λ approaching zero will indicate the split-complex nature. Since most real world signals change their nature (and statistics) with time, the proposed choice of relatively short length feedforward nonlinear structures, clearly has the potential to deal with this problem.

To preserve the convexity (and hence the existence of the solution) of the structure from Fig. 4, the parameter $\lambda(k)$ is kept real and is updated by optimising the same cost function as the two constitutive

[a] Since every convex problem has a unique solution, this choice of λ guarantees the solution to this optimisation problem.

subfilters. This way, using a stochastic gradient adaptation, we have

$$\lambda(k+1) = \lambda(k) - \mu_\lambda \nabla_\lambda E(k)_{|\lambda=\lambda(k)} \tag{7}$$

where μ_λ is the adaptation step size for the update of the mixing parameter $\lambda(k)$.

From (4) and (6), the gradient within the updates of the mixing parameter $\lambda(k)$ can be derived as

$$\nabla_\lambda E(k)_{|\lambda=\lambda(k)} = \frac{1}{2}\left\{ e(k)\frac{\partial e^*(k)}{\partial \lambda(k)} + e^*(k)\frac{\partial e(k)}{\partial \lambda(k)} \right\} \tag{8}$$

yielding the update of the mixing parameter λ in the form

$$\lambda(k+1) = \lambda(k) + \mu_\lambda \Re\{e(k)(y_{fully}(k) - y_{split}(k))^*\} \tag{9}$$

where $\Re(\cdot)$ denotes the real part of a complex quantity. This concludes the derivation of the learning algorithm for the hybrid (convex) nonlinear adaptive filter in the complex domain, shown in Fig. 4.

3.3. *Performance on synthetically generated data*

Let us illustrate the performance of the proposed online signal modality tracking algorithm on the example of the synthetically generated Ikeda map (chaotic signal),[30] given by

$$\begin{aligned} x(n+1) &= 1 + u\,[x(n)\cos t(n) - y(n)\sin t(n)] \\ y(n+1) &= u\,[x(n)\sin t(n) + y(n)\cos t(n)] \end{aligned} \tag{10}$$

where u is a parameter and

$$t(n) = 0.4 - \frac{6}{1 + x^2(n) + y^2(n)}.$$

Due to the signal generation mechanism in the form of a set of coupled partial difference equations, Ikeda map (10) represents a fully-complex signal. Following the results from,[31] we now conduct simulations using the hybrid configuration from Fig. 4; the results of the simulation are shown in Fig. 5.

As desired, the learning curve from Fig. 5 confirms the fully-complex nature of Ikeda map, as indicated by the values of mixing parameter $\lambda(k)$ being close to unity, hence facilitating the subfilter from Fig. 4, with a fully-complex nonlinear activation function.

From Fig. 4, since both the subfilters are stable individually, the hybrid filter will be stable by virtue of the convexity of the combination of the constituent subfilters.

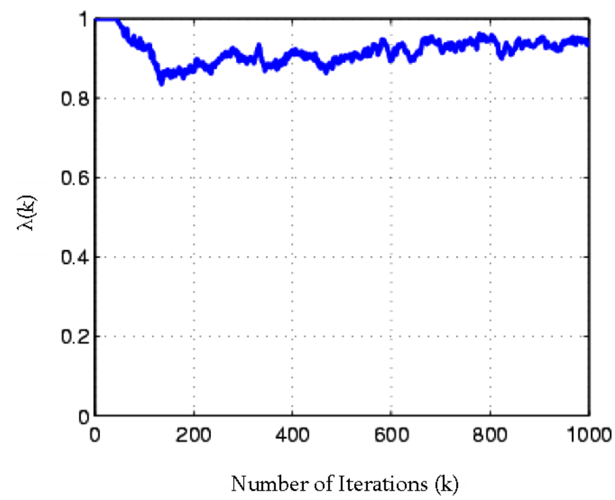


Fig. 5. Evolution of the mixing parameter λ for the prediction of Ikeda map.

4. Simulations

4.1. *Detection of the modality of sea clutter*

To validate the usefulness of the proposed approach on real world data, we first combine the experiments from Refs. 8 and 31 and apply our methodology to radar data coming from two very different settings. In Ref. 8, it was shown (surrogate data based test for the complex nature of the data) that the nature of radar data from a maritime radar (IPIX, publicly available from Prof. Simon Haykin's web site) change with the recording conditions:

- Sea clutter is fully-complex when the sea state is "high" (more turbulences) or when the sea state is "low" and the ocean waves are coming toward the radar;
- Sea clutter is split-complex when the sea state is "low" and the waves are moving away from the radar.

To ascertain whether we can track the signal nature using the proposed model, we synthetically generate the input by alternating blocks of 400 samples recorded during one or the other sea state. The resulting evolution of λ at the output of the hybrid filter is shown in Fig. 6.

The time variation of the mixing parameter $\lambda(k)$ confirms that the modality of radar data recorded during a "high" sea state with waves coming toward the radar was predominantly fully-complex, as illustrated by the values of $\lambda(k)$ being close to

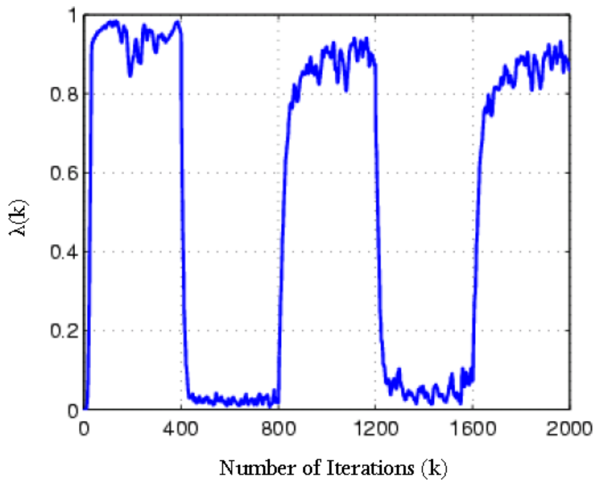


Fig. 6. Evolution of the mixing parameter $\lambda(k)$ on the prediction of a signal generated by altering blocks of 400 data samples of the radar data from the "low" and "high" sea state.

unity and consequently favouring the fully-complex subfilter from Fig. 4. Conversely, the modality of the radar data coming from the other recording conditions was predominantly split-complex. The combination was thus clearly capable of identifying the nature of radar data; which is critical for the correct interpretation of the readings obtained from such a device (e.g. existence of a target).

4.2. *Detection of the modality of wind data*

It has been shown in Sec. 2.2 that wind can be considered as a complex-valued quantity. The modeling of wind is very demanding, and some results on nonparametric tests for the complex nature of this signal are given in the Appendix. Next, in order to examine the usefulness of the proposed approach for the processing of real world signals, a set of wind measurements was input to the structure from Fig. 4, which allowed us to determine the split-/fully-complex nature of the data set. The measurements were started at about 2.00 pm; lasted for approximately 24 hours, at a sampling rate of 1Hz and were recorded in an urban environment.

For the experiments, the learning rates of the split- and fully-complex Nonlinear Gradient Descent algorithms were set to $\mu_{split} = \mu_{fully} = 0.01$. The step-size of the convex combination was set to $\mu_\lambda = 0.5$, while the filter length was $N = 10$. Figure 7 shows that the wind signal is mainly fully-complex,

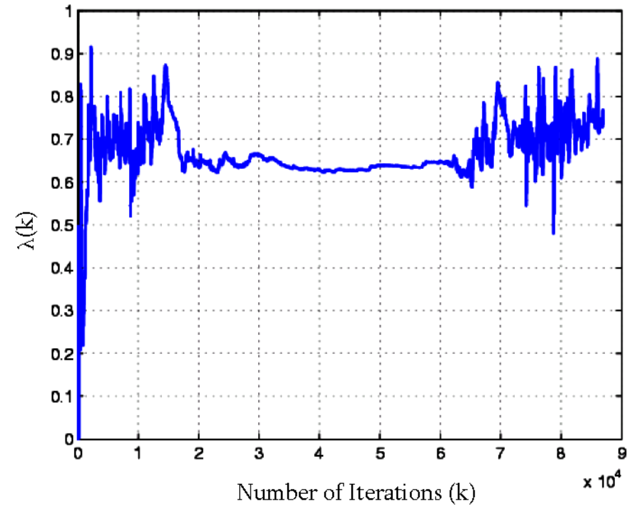


Fig. 7. Evolution of the mixing parameter λ for the prediction of wind during the period of one day.

since $\lambda(k)$ was centered around 0.65 the majority of the time. From 2.00 pm until 6.00 pm (samples 1–2000) and from 8.00 am till 2.00 pm (samples 6000–8500) the next day, the type of nonlinearity in the signal is varying, as indicated by the oscillation of λ in the range $[0.5, 0.9]$. On the contrary, during the late evening and night periods (samples 2000–6000), which are "calmer", λ remained constant, showing that the type of nonlinearity did not change. This also conformed to our recent results on wind modeling for renewable energy applications.[29]

5. Conclusions

We have introduced a novel methodology for discriminating between the split- and fully-complex nature of real world complex-valued signals. This has been achieved based on a convex combination of nonlinear complex-valued adaptive filters, trained respectively by a split-complex and fully-complex nonlinear gradient descent. The convexity constraint ensures the existence of the solution, and the value of the convex mixing parameter within this hybrid filter indicates the predominant nature (modality) of a signal in hand. Simulations on both benchmark and real world data support the proposed approach.

Acknowledgments

We would like to thank Prof. Kazuyiki Aihara and Dr Yoshito Hirata for fruitful discussions and for providing us with the wind data sets.

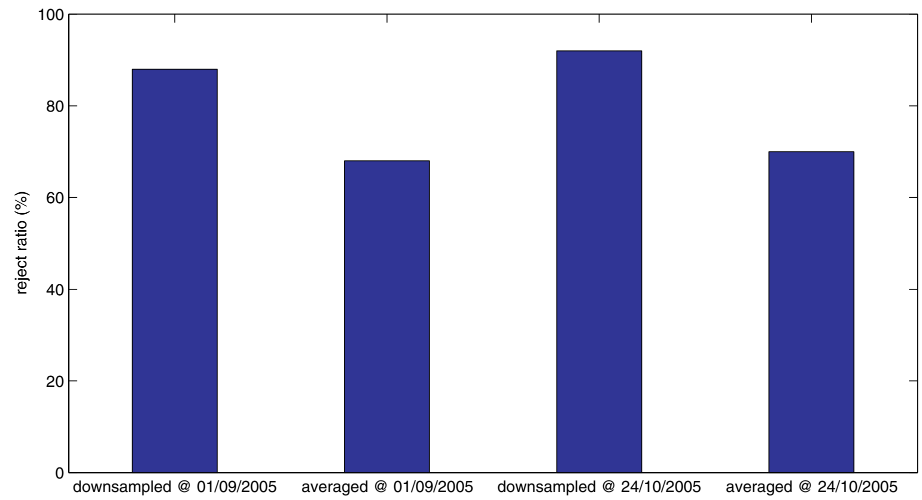


Fig. 8. The complex-valued surrogate data test for the complex nature of wind signal (speed and direction).

Appendix: Nonparametric Testing for the Complex Nature of Wind

In order to support the intuitive indication of a complex-valued nature of wind and following the approach from Ref. 8, a component dependent test for the complex-valued wind representation was performed. This method is set within the framework of hypothesis testing and the statistical testing methodology from[14,3] is adopted. The null hypothesis is that the original signal is complex-valued and the signals are characterised by Delay Vector Variance (DVV) method.

The test is based on the complex-valued surrogate data analysis and indicates that there is a significant component dependence within the complex-valued wind signal. Figure 8 shows the results of this test on two set of wind data recorded in an urban environment and obtained from the Institute of Industrial Science, University of Tokyo, Japan. The number of surrogates used amounts to 100 and the % rejection of the null hypothesis is plotted for wind signals averaged over either one hour intervals or six hour intervals. From the figure, the rejection ratio of the null hypothesis of fully-complex wind data is significantly greater than zero, thus indicating the need for a simultaneous treatment of the speed and direction components of wind. Furthermore, as can be seen from the rejection ratios, the signals averaged over one hour show a stronger indication of having a complex nature than those averaged over six hours. Therefore complex-valued wind signals become more univariate and linear when averaged over longer intervals, this is in line with results from probability theory where a random signal becomes more Gaussian (and therefore linear) an the increase in the degree of averaging.

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