Adaptive Plenoptic Sampling

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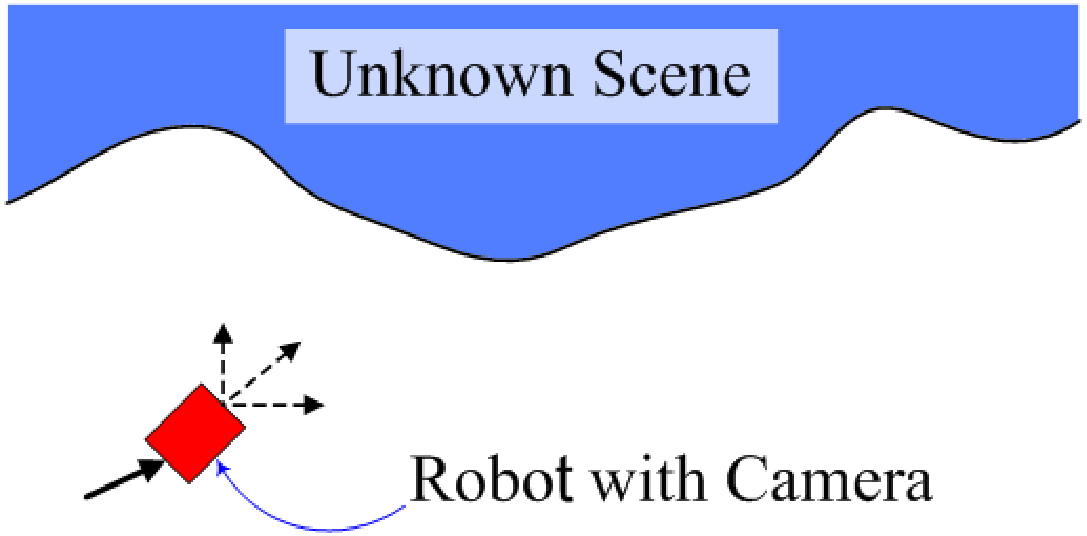
Motivation

Consider an unknown scene with a certain texture, in order to render good quality viewpoint, for Image-Based Rendering (IBR), the scene must be adequately sampled. Suppose the sampling can be achieved using a camera mounted to a robot.



Important Questions:

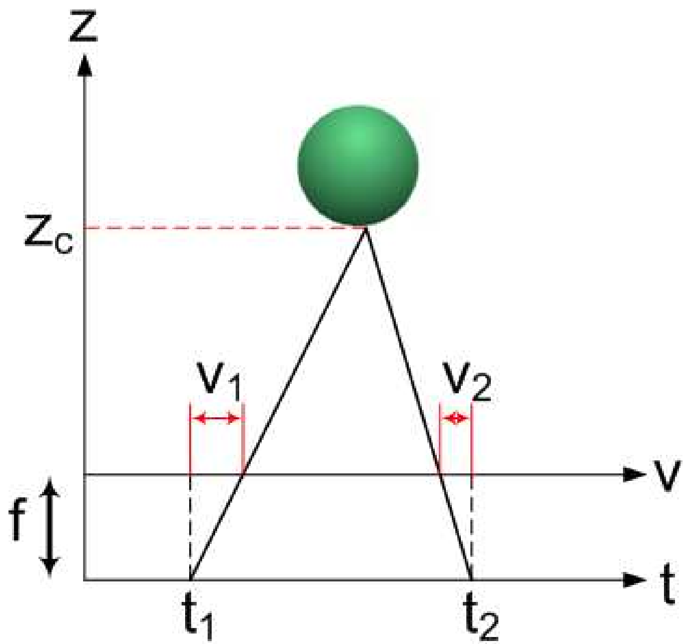
- How should the camera travel relative to the scene?
- Where to sample along the path?
- Whether to zoom or not?

Current Work:

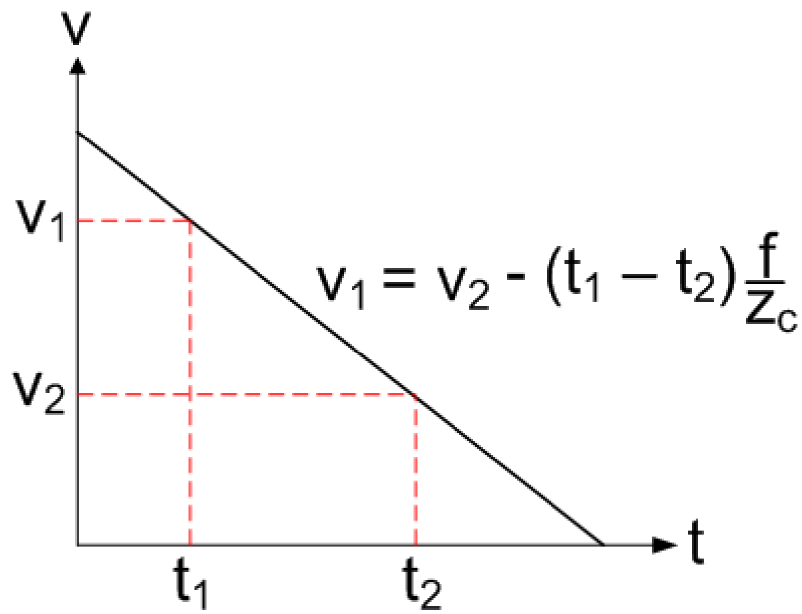
- Approximating the scene with a linear depth model and bandlimited texture
- Bandwidth analysis of such a model to determine maximum uniform camera spacing

Plenoptic Function and the Epipolar Plane Image

Consider the 2D Plenoptic Function, $p(t, v)$, known as the Epipolar Plane Image (EPI) [2]



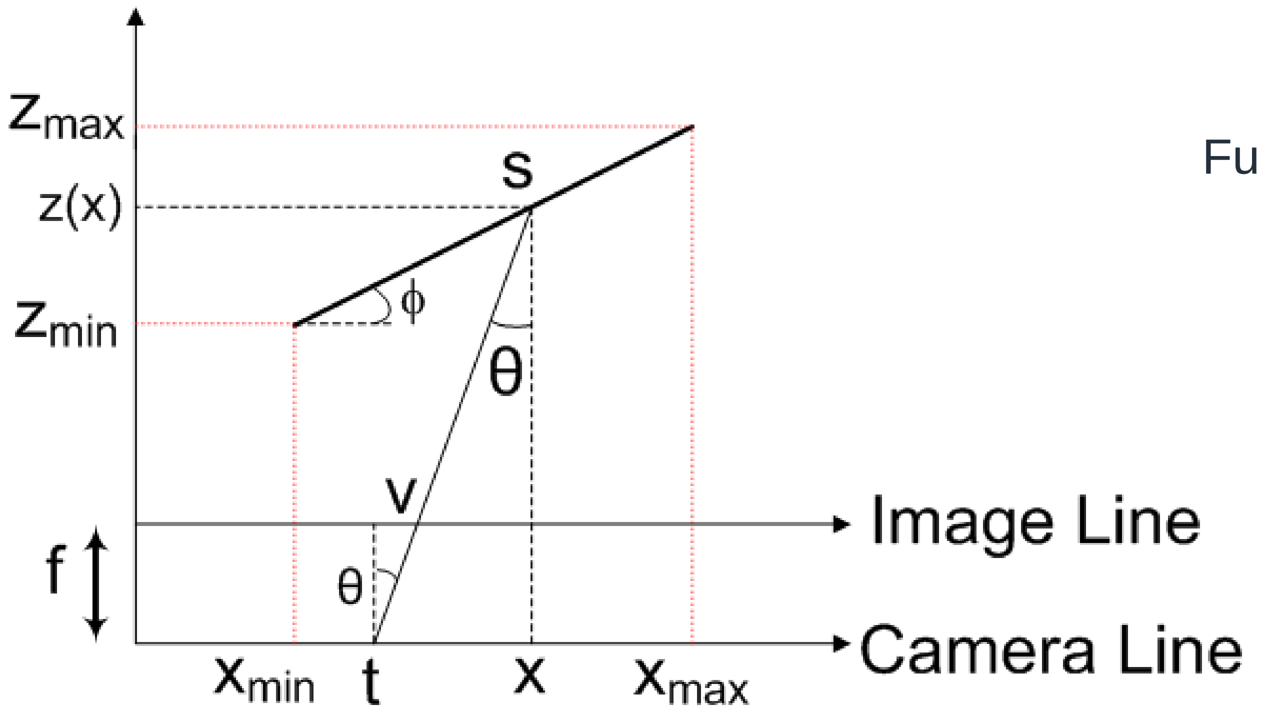
(a) Scene



(b) EPI

- Point in the scene $\Longrightarrow$ Line in the EPI plane where the slope depends on the depth
- Fixing a camera position t_1 $\Longrightarrow$ 1D image signal
- Fixing a pixel v_1 $\Longrightarrow$ 1D signal of the pixel captured by all cameras

Slanted Plane Geometry



Functional Scene Model [3]:

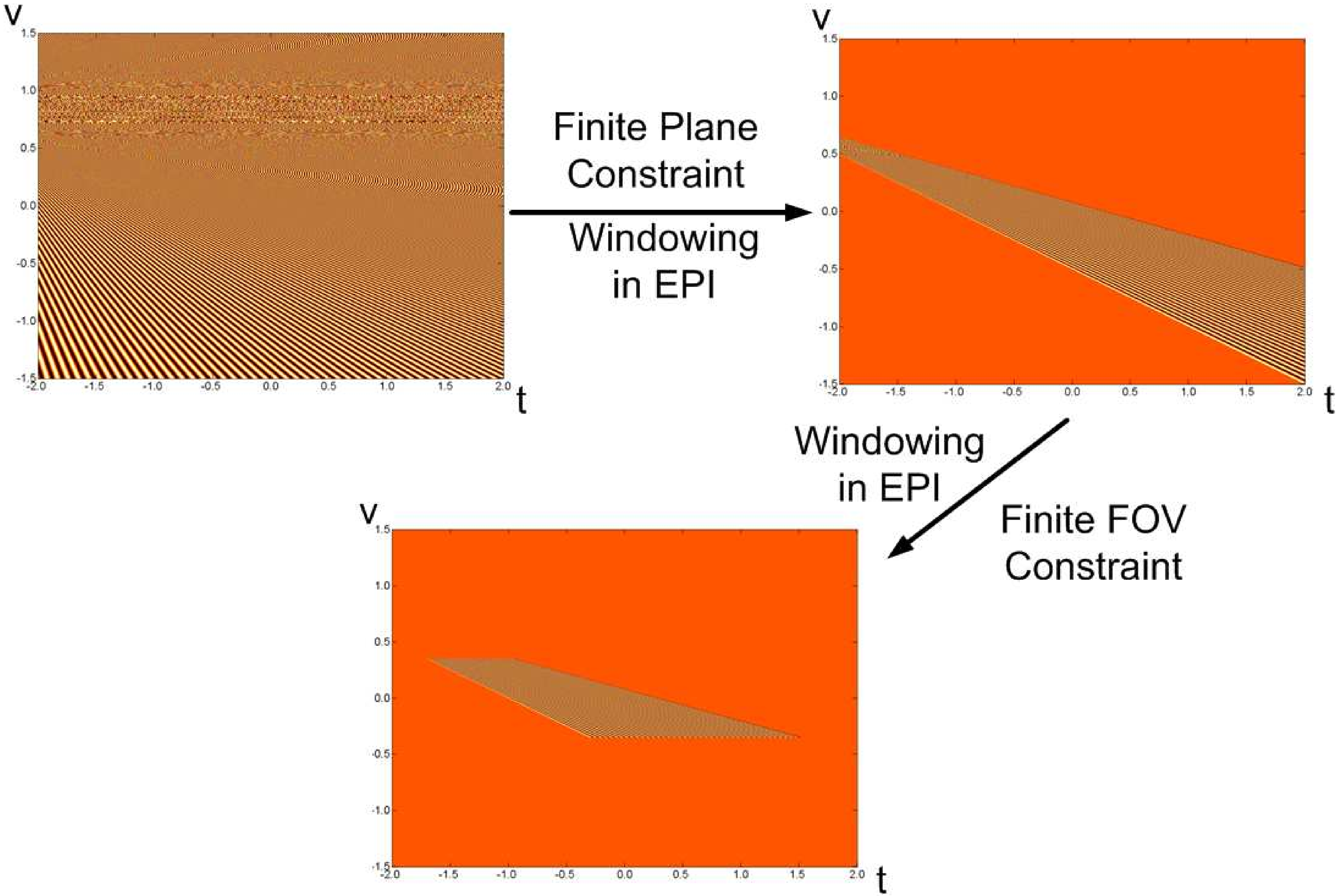
- s is the Curvilinear Coordinate
- x is the Projection of s onto t
- ϕ is the Slant of the Plane

Texture Signal Pasted to Scene Surface, $g(s) = \sin(\omega_s s)$

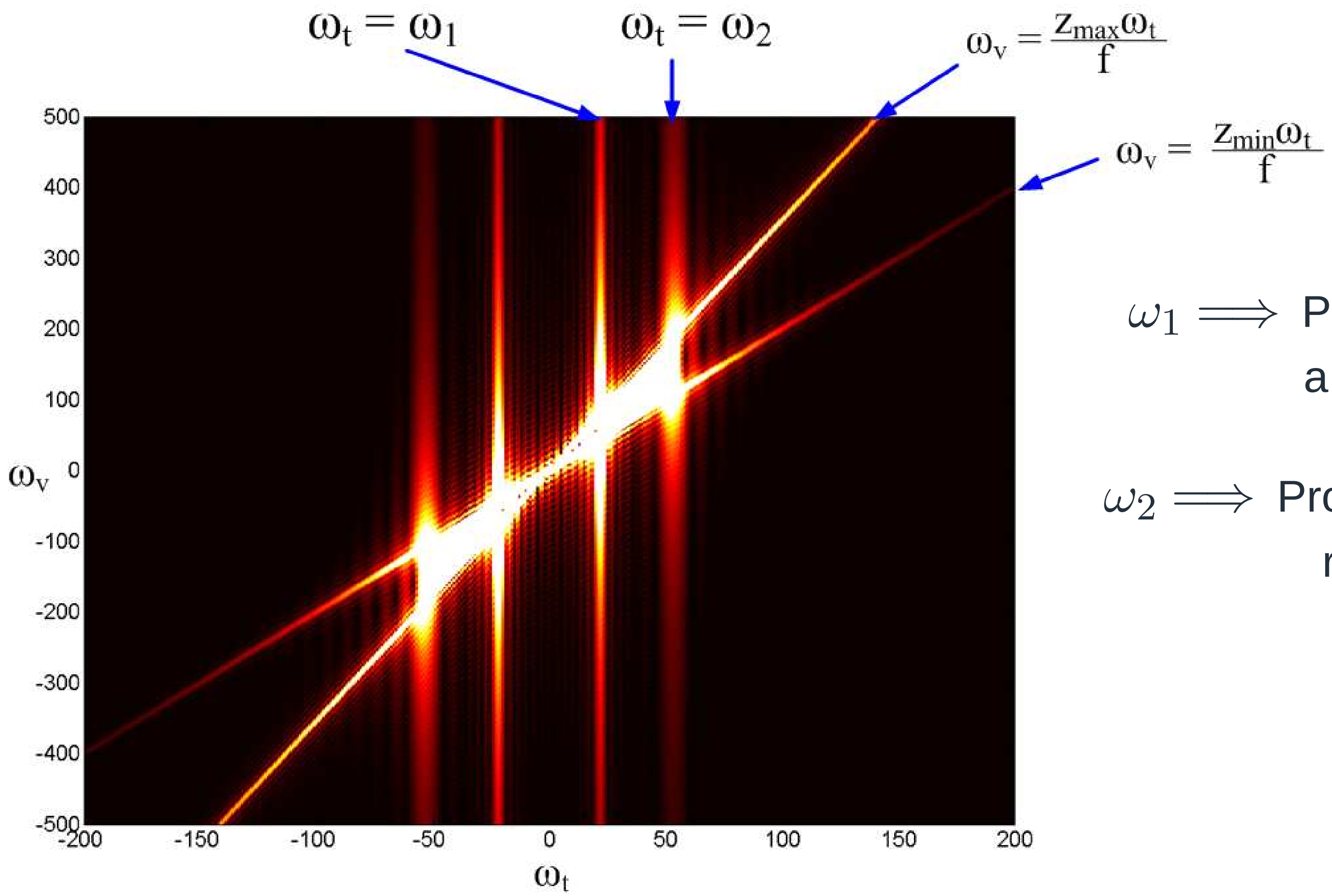
Constraints:

- Finite Field of View (FoV) for the Cameras $\Longrightarrow v \in [-v_{max}, v_{max}]$
- Finite Plane Width $\Longrightarrow s \in [0, T_{geo}]$
- Lambertian Scene

Effect of the Constraints on the EPI



Slanted Plane Plenoptic Spectrum



$\omega_1 \Longrightarrow$ Projected frequency from a ray at $-v_{max}$

$\omega_2 \Longrightarrow$ Projected frequency from a ray at $+v_{max}$

- Finite Plane Constraint $\Longrightarrow$ Spectral spreading along lines relating to z_{max} and z_{min}
- Finite FoV Constraint $\Longrightarrow$ Spectral spreading in the ω_v direction, for $\omega_t = \pm\omega_1$ and $\omega_t = \pm\omega_2$

Evaluating Plenoptic Spectrum for $g(s) = e^{j\omega_s s}$

The Plenoptic Spectrum

$$|P| = \left| \frac{\omega_s f}{\sin(\phi)\omega_t^2} \left[\zeta(jb(c-1)) - \zeta(ja(c-1)) - \zeta(jb(c+1)) + \zeta(ja(c+1))\right] + \frac{2v_{max}}{\omega_t} \left[\text{sinc}(a)\, e^{-jca} - \text{sinc}(b)\, e^{-jcb}\right] \right|$$

where

$$a = \omega_v v_{max} - \omega_t \frac{z_{max} v_{max}}{f} \;,\; b = \omega_v v_{max} - \omega_t \frac{z_{min} v_{max}}{f} \;,\; c = \frac{-f(\omega_t \cos(\phi) - \omega_s}{\sin(\phi)\omega_t v_{max}}$$

and $\zeta(jx)$, for $x \in \mathbb{R}$, is defined as

$$\zeta(jx) = \begin{cases} \text{E}_1(jx) + \ln(jx) + \gamma & \text{, if } x > 0 \\ \text{E}_1(-jx) - 2j\text{Si}(-x) + j\pi + \ln(jx) + \gamma & \text{, if } x < 0 \\ 0 & \text{, if } x = 0 \end{cases}$$

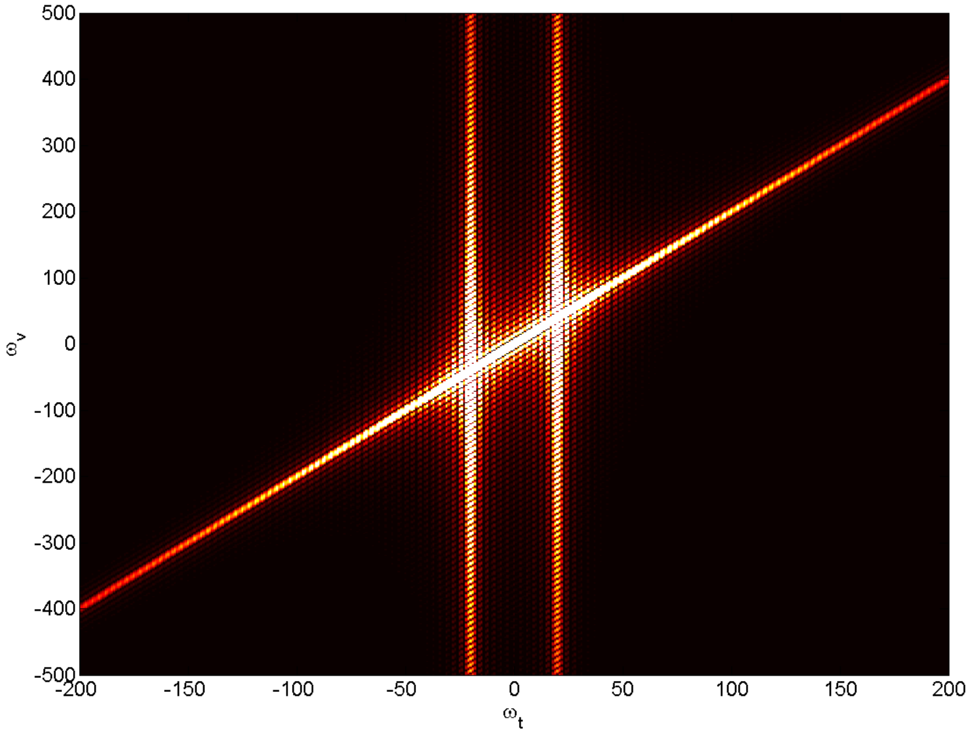
$\text{E}_1(w)$ is the Exponential Integral, $\text{Si}(w)$ is the Sine Integral and γ is Euler's Constant [1]

Note that for $\omega_t = 0$ the plenoptic spectrum is

$$|P| = \text{sinc}\left(\frac{T_{geo}\omega_s}{2}\right) \left| \cos(\phi)\text{sinc}(\omega_v v_{max}) - j\sin(\phi)\left(\frac{\cos(\omega_v v_{max})}{f\omega_v} - \frac{\sin(\omega_v v_{max})}{f v_{max} \omega_v^2}\right) \right|$$

Bandwidth Analysis

Consider a special case, when $\phi = 0$, $\implies$ Flat frontal parallel plane at a depth z_c



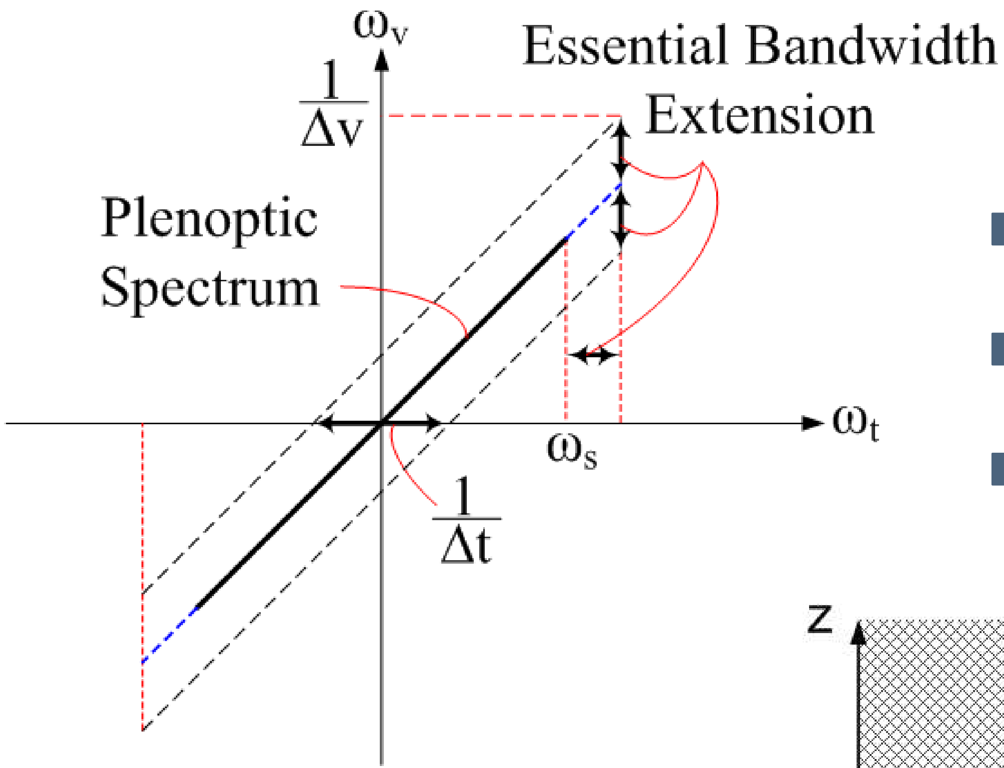
Spectrum non-bandlimited in either ω_v or ω_t

- Convolution with Sinc function along the line $\omega_v = \frac{\omega_t z_c}{f}$
- Convolution with Sinc function parallel to ω_v-axis at $\omega_t = \pm\omega_s$

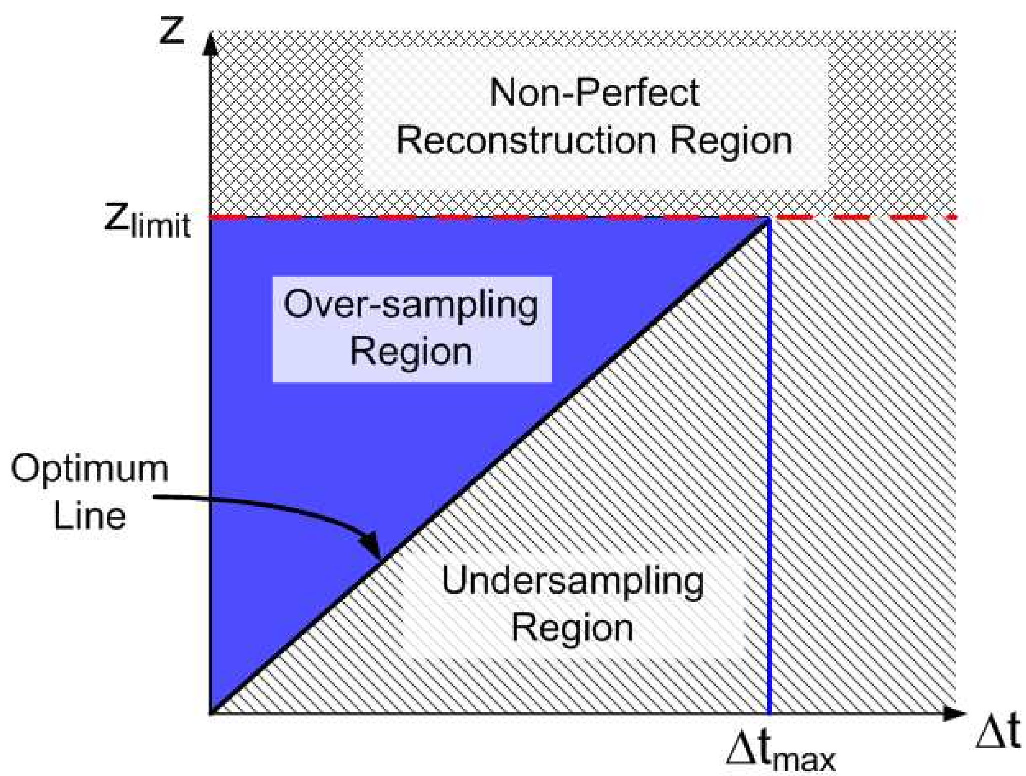
Finite bandwidth covering 90% of the signal's energy $\implies$ Essential Bandwidth [4]

For a Sinc function $\implies$ Essential Bandwidth is the main lobe [4]

Sampling Curve



- $\Delta t \implies$ Spacing between Cameras
- $\Delta v \implies$ Finite Pixel Resolution
- Optimum Line $\implies$ Optimum Δt for given z



Conclusions and References

- The finite constraints imposed lead to spectral spreading in the ω_v-ω_t domain. Thus the plenoptic spectrum is no longer bound between the lines relating to minimum and maximum depth.
- However the plenoptic spectrum for a slanted plane with a sine wave texture can be expressed in a closed form expression.
- Simplifying to a frontal parallel plane, the finite constraints lead to convolution with sinc functions. Thus the plenoptic spectrum is not bandlimited in either ω_t or ω_v but the essential bandwidth can be defined.
- Using this essential bandwidth an optimum Δt can be derived for a given distance between the camera line and the scene. Plotting this optimum relationship leads to a sampling curve, which determines the optimum Δt given the depth.

References:

1. M. Abramowitz and I.A. Stegun. *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables.* Dover, 1964.

2. J.X. Chai, S.C. Chan, H.Y. Shum, and X. Tong. Plenoptic sampling. In Computer graphics (SIGGRAPH'00), pages 307-318, 2000.

3. M.N. Do, D Marchand-Maillet, and M. Vetterli. On the bandwidth of the plenoptic function. IEEE Transactions on Image Processing, 2009. Preprint.

4. B.P Lathi. *Modern Digital and Analog Communication Systems.* Oxford University Press, 1998.