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A Closed-Form Expression For the Bandwidth of the Plenoptic Function Under Finite Field of View Constraints

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Motivation

Consider an unknown scene with a certain texture pasted to the surface. In order to render good quality new viewpoints of the scene, using Image-Based Rendering (IBR) techniques, it must be adequately sampled. Suppose the sampling device is a camera mounted to a robot.

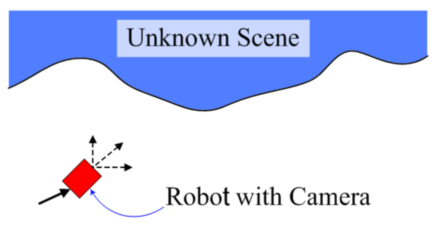

Fig 1: Unknown Scene

Important Questions:

- How should the camera travel relative to the scene?
- Where to sample along the path?
- Whether to zoom or not?

Our Approach:

- Approximate the scene with a planar facet model and bandlimited texture.
- Bandwidth analysis of the plenoptic function for such a model under the constraints of finite field of view and finite plane width.
- Use analysis to determine a non-uniform distribution for the camera along a 1D path parallel to the scene.

The Plenoptic Function

Consider the 2D plenoptic function[1], $p(t, v)$:

- Models the intensity of the light ray, travelling from the scene, at camera location t and pixel location v.
- Representation in the (t, v)-space is known as the Epipolar Plane Image (EPI), where a point in the scene is mapped to a line with a slope depending on its depth.
- The Fourier transform of the EPI gives the Plenoptic Spectrum, $P(\omega_t, \omega_v)$. The spectrum is bounded by lines relating to the maximum and minimum depths of the scene[2].

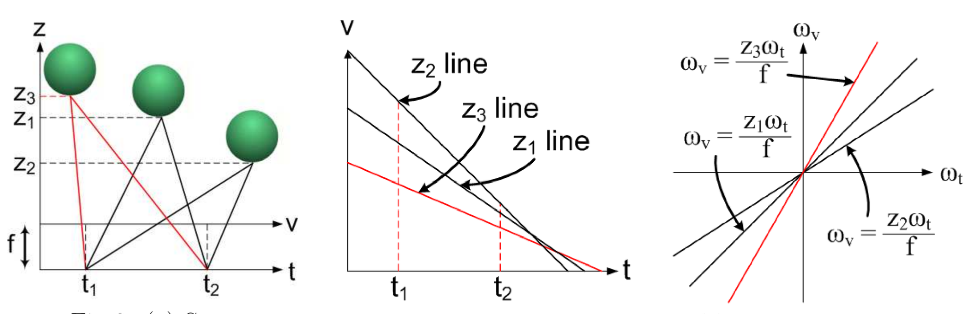

Fig 2: (a) Scene (b) EPI (c) Plenoptic Spectrum

Sampled Plenoptic Spectrum:

- Lowpass Filtering in ω_v due to finite resolution, Δv
- Finite camera spacing leading to replicated spectra.
- Undersampling results in spectral overlap (aliasing).

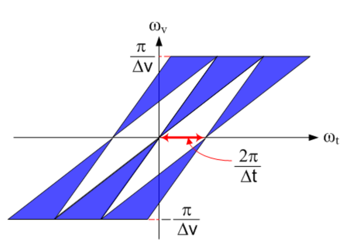

Fig 3: Sampled Plenoptic Spectrum

Slanted Plane Geometry

The scene is modelled using functional surfaces with bandlimited texture pasted to the surface[3]. In this case the texture signal is a sinusoid.

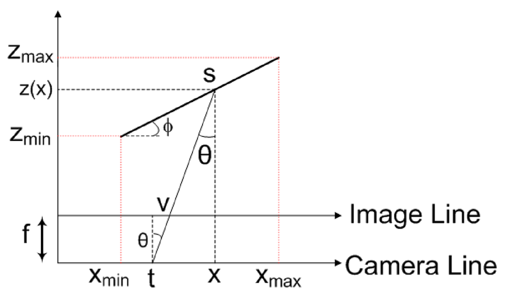

Fig 4: Scene Geometry

Parameters:

- s is the coordinate on the plane.
- x is the projection of s onto t.
- ϕ is the slant of the plane.
- θ is the angle of view.
- f is the focal length.

Surface light field relationship:

$$l(x, \theta) = p(t, v), \text{ when } t = x - z(x)\frac{v}{f}$$

Constraints:

- Finite Field of View (FoV) $\Longrightarrow v \in [-v_m, v_m]$
- Finite Plane Width $\Longrightarrow s \in [0, T]$
- Lambertian Surface $\Longrightarrow l(x, \theta) = l(x)$

Slanted Plane Analysis

Applying both finite constraints to the plenoptic function of the slanted plane lead to windowing in the EPI domain, shown below:

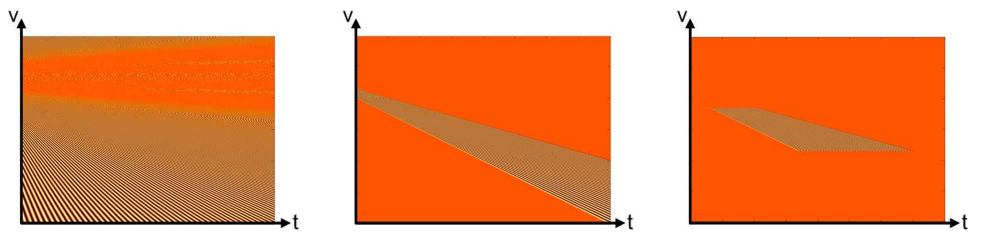

Fig 5: (a) Unconstrained EPI (b) Finite Plane Width EPI (c) Finite FoV EPI

Plenoptic Spectrum

The plenoptic spectrum is derived by taking the Fourier transform of the finite EPI in Fig 3(c), mathematically this is defined by the integral:

$$P(\omega_t, \omega_v) = \int_{s=0}^{s=T} g(s)\cos(\phi) \int_{v=-v_m}^{v=v_m} \left[1 - v\frac{\tan(\phi)}{f}\right] e^{-j(\omega_v - s\frac{\sin(\phi)\omega_t}{f})v} e^{-j\omega_t \cos(\phi) s}\, dv ds$$

The solution to this integral, when $g(s) = e^{j\omega_s s}$, is

$$|P(\omega_t, \omega_v)| = \left| \frac{\omega_s f}{\sin(\phi)\omega_t^2} \left[\zeta(jb(c-1)) - \zeta(ja(c-1)) - \zeta(jb(c+1)) + \zeta(ja(c+1))\right] + \frac{2v_m}{\omega_t}\left[\operatorname{sinc}(a)\, e^{-jca} - \operatorname{sinc}(b)\, e^{-jcb}\right] \right|$$

Where $\zeta(jw)$, for $w \in \mathbb{R}$, is defined as

$$\zeta(jw) = \begin{cases} \mathrm{E}_1(jw) + \ln(jw) + \gamma & , \text{ if } w > 0 \\ \mathrm{E}_1(-jw) - 2j\mathrm{Si}(-w) + j\pi + \ln(jw) + \gamma & , \text{ if } w < 0 \\ 0 & , \text{ if } w = 0 \end{cases}$$

$\mathrm{E}_1(w)$ is the Exponential Integral, $\mathrm{Si}(w)$ is the Sine Integral and γ is Euler's Constant [4] and

$$a = \omega_v v_m - \omega_t \frac{z_{max} v_m}{f}, \quad b = \omega_v v_m - \omega_t \frac{z_{min} v_m}{f}, \quad c = \frac{-f(\omega_t\cos(\phi) - \omega_s)}{\sin(\phi)\omega_t v_m}$$

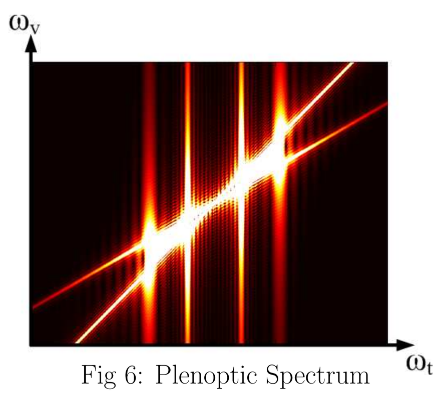

Fig 6: Plenoptic Spectrum

The Essential Bandwidth

The plenoptic spectrum of the slanted plane is band-unlimited in both ω_v and ω_t, hence we cannot define an exact bandwidth region. However, we can define the Essential Bandwidth[5]:

A finite region that contains 90% of the signal's energy.

Central Concept: The plenoptic function is adequately reconstructed when it is assumed to be bandlimited to the essential bandwidth.

Essential Bandwidth Model

The essential bandwidth is parameterised using four parameters:

- Ω_t is the maximum value in ω_t,
- Ω_v is the maximum value in ω_v,
- z_{opt}/f is the slant of region,
- A is the width of region in ω_t.

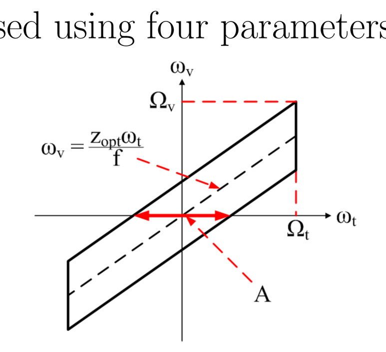

Fig 7: Essential Bandwidth Model

Applied to the slanted plane:

- Restrict the degrees of freedom so Ω_t is the only free parameter

$$\text{Fix:} \quad \Omega_v = \Omega_t \frac{z_{max}}{f} \quad \text{and} \quad z_{opt} = \frac{2}{z_{max}^{-1} + z_{min}^{-1}} \quad \Rightarrow \quad A = \Omega_t \frac{T\sin(\phi)}{z_{min}}$$

- Assume the plenoptic spectrum is characterised as shown in Fig 8(a).
- Approximate the decay along each line outside the region and equate the total energy to 10%

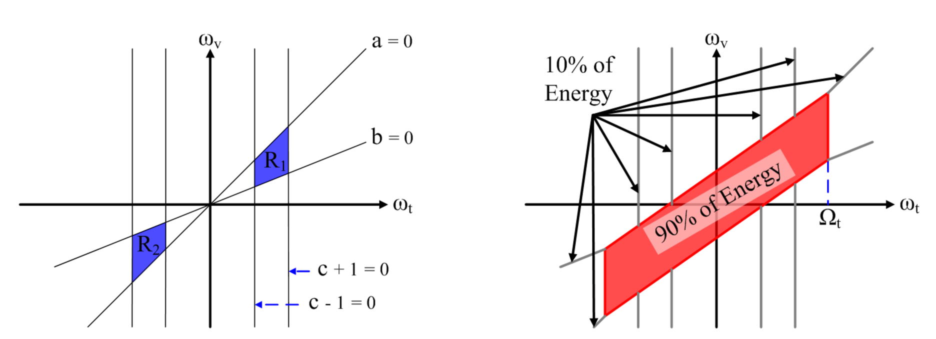

Fig 8: (a) Characterisation of Spectrum (b) Calculating Ω_t

Fronto-Parallel Plane

A special case when $\phi = 0$, a finite fronto-parallel plane (FPP) at a depth z_c. The plenoptic spectrum is now:

$$|P(\omega_t, \omega_v)| = \operatorname{sinc}\left(\omega_v v_m - \frac{z(x)v_m}{f}\omega_t\right) \left| G(\omega_t) * \operatorname{sinc}\left(\frac{\omega_t T}{2}\right) e^{-j\omega_t \frac{T}{2}} \right|$$

where $G(\omega_t)$ is the Fourier transform of $g(s)$, bandlimited to ω_s.

- Finite Plane Width $\Rightarrow$ Convolution with Sinc function in spectral domain along the line $\omega_v = \omega_t \frac{z_c}{f}$.
- Finite FoV $\Rightarrow$ Convolution with Sinc function in spectral domain parallel to the ω_v-axis.

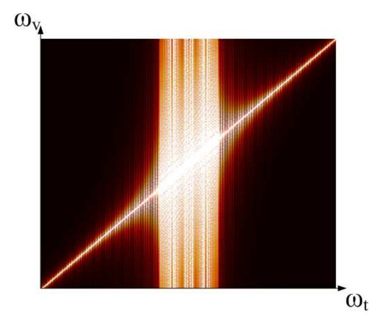

Fig 9: Plenoptic Spectrum for a FPP

Sampling and Reconstruction

The essential bandwidth of a Sinc function is the width of its main lobe, thus the essential bandwidth parameters for a FPP are:

$$\Omega_t = \omega_s + \frac{2\pi}{T}, \quad \Omega_v = \Omega_t \frac{z_c}{f} + \frac{\pi}{v_m}, \quad z_{opt} = z_c \quad \text{and} \quad A = \frac{2\pi f}{v_m z_c}$$

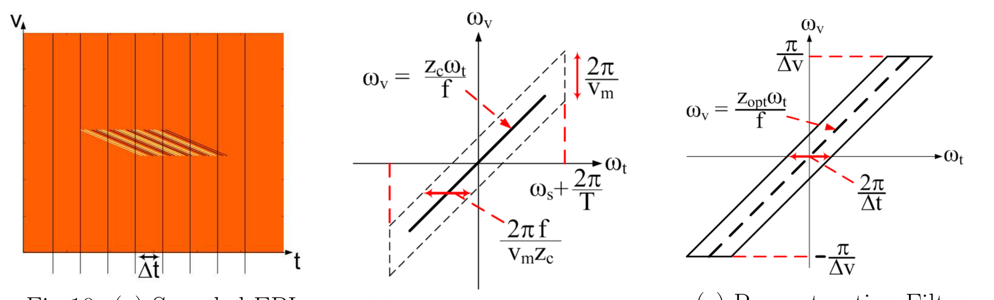

Fig 10: (a) Sampled EPI (b) Essential Bandwidth (c) Reconstruction Filter

Critical sampling determined by relating the essential bandwidth region to the reconstruction filter:

$$\Delta t = \frac{v_m z_c}{f}$$

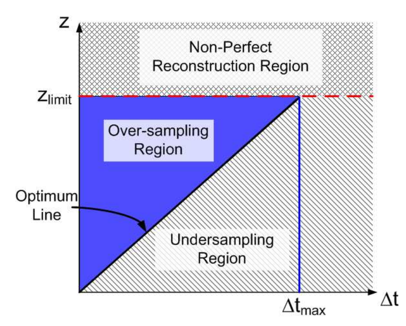

Fig 11: Sampling Curve

Sampling Curve:

- Adequate camera spacing given the distance from the cameras to the scene.
- The camera spacing can be increased by moving further from the scene.
- Oversampling occurs to left of the optimum line and undersampling to the right.

Simulation Results

A scene consisting of two identical FPPs at different depths.

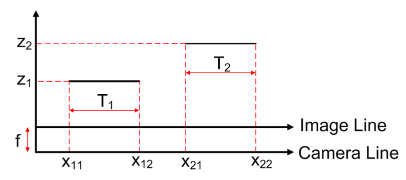

Fig 12: Two Plane Geometry

Uniform Sampling $\Longrightarrow$ Sample rate dictated by nearest plane, thus other plane is oversampled

Scene is piecewise planar $\Longrightarrow$ Non-uniform sampling scheme

Sample rate varies depending on the plane being sampled.

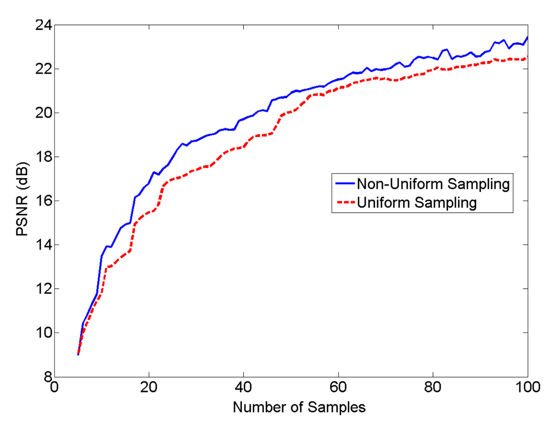

Fig 13: Simulation Results

References

1. E.H. Adelson and J.R. Bergen. The plenoptic function and the elements of early vision. In Computational Models of Visual Processing, pages 3-20. MIT Press, Cambridge, MA, 1991.
2. J.X. Chai, S.C. Chan, H.Y. Shum, and X. Tong. Plenoptic sampling. In Computer graphics (SIGGRAPH'00), pages 307-318, 2000.
3. M.N. Do, D Marchand-Maillet, and M. Vetterli. On the bandwidth of the plenoptic function. IEEE Transactions on Image Processing, 2009. Preprint.
4. M. Abramowitz and I.A. Stegun. *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*. Dover, 1964.
5. B.P Lathi. *Modern Digital and Analog Communication Systems*. Oxford University Press, 1998.