LETTER

Linear Precoding Based on Sub-Channel Permutation in Post-Combining MIMO-HARQ Systems*

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SUMMARY A powerful HARQ-based linear precoding scheme is proposed to utilize the flexibility of post-combining HARQ strategy in MIMO communications systems. The scheme selects the appropriate symbols and transmit powers for each eigen-mode to acquire more performance gains. Simulation results show that the proposed scheme achieves about 5.5 dB signal-to-noise ratio gains over original spatial multiplexing scheme at an average bit error rate of 10^{-4}. Furthermore, the gap between the two schemes increases with the number of transmissions.

key words: *MIMO, HARQ, linear precoding, MMSE, ZF, channel state information*

1. Introduction

Recently, with the rapid development of the demand of reliable service of high data rate, hybrid automatic repeat request (HARQ) protocols are becoming one of the key technologies in B3G mobile communications systems. In comparison to ARQ techniques [1], HARQ combines all the retransmission data to achieve higher decoding reliability and throughput instead of abandoning the erroneous data. Both chase combining and incremental redundancy [2] are primary schemes for data combinations. The former method requires relatively smaller buffer size at the receiver with lower cost of the system; while the latter one is a bit-level combination with larger buffer size and additional signaling, but it achieve more performance gains.

On the other hand, multiple antennas could be established at both transmitter and receiver to achieve higher frequency efficiency and link reliability. [3] proposed two combining methods for multiple-input multiple-output (MIMO) systems, termed pre-combining and post-combining. The pre-combining scheme outperforms post-combining for a given receiver structure. However, it is a symbol-level combining scheme and only applicable when the same symbols are retransmitted, while post-combining is more flexible and combinations could be performed at both symbol-level and bit-level. In Ref. [8], a HARQ scheme was proposed to increase the free distance by using various trellis-coded modulation codes for retransmissions. Antenna permutation was also adopted to take full advantage of the flexibility of MIMO systems. The previous work of HARQ did not consider channel state information (CSI) at both ends. However, linear procoding technologies [4], [7] enhance the system performance significantly when the transmitter knows partial or full CSI. Generally, the transmitter could acquire the CSI either through the feedback channel in frequency division duplexing mode, or through the channel reciprocal property between the uplink and downlink in time division duplexing mode. In this paper, we propose a novel linear precoding scheme for post-combining HARQ system with full CSI at the transmitter. Simulations confirm the superiority of our scheme over the traditional spatial multiplexing and additive linear precoding schemes.

2. System Model

We firstly describe a post-combining HARQ structure in MIMO systems. At the i-th data transmission, the input bit stream is coded and modulated to generate symbol vector $\mathbf{s}_{(i)}$. The output is then passed through the linear precoder matrix $\mathbf{F}_{(i)}$ and launched into the MIMO channel through the M transmit antennas. At the receiver, the baseband signals from N receive antennas are detected by linear receiver $\mathbf{G}_{(i)}$ and the output spatial symbols $\tilde{\mathbf{s}}_{(i)}$ are then processed by the demodulator and decoder. If a decoding error occurs, the symbol vector should be passed through the new precoder matrix $\mathbf{F}_{(i+1)}$ in the next loop after the transmitter receives the indicating signals for retransmission from the receiver. The receiver would then be updated to $\mathbf{G}_{(i+1)}$ and chase combining would be performed either on bit-level or symbol-level. The same procedure will be carried on until there is no error or it reaches a maximum of L transmissions.

The system equation for the i-th transmission is

$$\tilde{\mathbf{s}}_{(i)} = \mathbf{G}_{(i)}\mathbf{H}_{(i)}\mathbf{F}_{(i)}\mathbf{s}_{(i)} + \mathbf{G}_{(i)}\mathbf{n}_{(i)}, \tag{1}$$

where $\mathbf{s}_{(i)} = [s_1, s_2, \cdots, s_B]^T$ is the $B \times 1$ transmitted vector, $(\cdot)^T$ denotes the transpose and $B \leq \min(M, N)$. $\mathbf{F}_{(i)}$ is the $M \times B$ precoder matrix which is optimized according to each channel realization and L. $\mathbf{H}_{(i)}$ is the $N \times M$ channel matrix whose entries are i.i.d zero-mean complex Gaussian variables with variance σ_h^2. $\mathbf{n}_{(i)}$ is the complex Gaussian noise vector with distribution of zero mean and σ_n^2 variance.

In this letter, linear receiver $\mathbf{G}_{(i)}$ is considered, which mainly includes minimum mean-square error (MMSE) receiver and zero-forcing (ZF) receiver. For MMSE receiver, $\mathbf{G}_{(i)}$ is given by $\mathbf{G}_{(i)} = (\mathbf{F}_{(i)}^H\mathbf{H}_{(i)}^H\mathbf{H}_{(i)}\mathbf{F}_{(i)} + \sigma_n^2\mathbf{I}_B)^{-1}\mathbf{F}_{(i)}^H\mathbf{H}_{(i)}^H$; for ZF receiver, $\mathbf{G}_{(i)} = (\mathbf{F}_{(i)}^H\mathbf{H}_{(i)}^H\mathbf{H}_{(i)}\mathbf{F}_{(i)})^{-1}\mathbf{F}_{(i)}^H\mathbf{H}_{(i)}^H$, where $(\cdot)^H$ denotes the conjugate transpose and $\mathbf{I}_B$ is the $B \times B$ identity

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matrix.

3. Linear Precoding Based on Post-Combining HARQ Scheme

3.1 Optimal Linear Precoding Scheme in Non-HARQ Strategy

In [4], [5], optimal linear precoding schemes were discussed for several criteria where both the transmitter and receiver know ideal CSI. Assuming that the modulated symbols are normalized to the unit power with the total transmit power P_0 constrained by $\mathrm{Tr}(\mathbf{F}\mathbf{F}^H) \le P_0$, we focus on optimizing the linear precoder $\mathbf{F}$ corresponding to each specific channel matrix, the type of the receiver and design criterion, where $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix. When MMSE criterion is adopted, the optimal precoders for both MMSE and ZF receivers are given by $\mathbf{F}_{MMSE} = \mathbf{V}\boldsymbol{\Phi}_{MMSE}$ and $\mathbf{F}_{ZF} = \mathbf{V}\boldsymbol{\Phi}_{ZF}$, where $\mathbf{V}$ is an orthogonal matrix which forms a basis for the range space of $\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}$ and $\mathbf{R}_n$ is the noise covariance matrix. If we carry on the eigenvalue decomposition, $\boldsymbol{\Lambda}$ is the diagonal matrix which contains the B most largest non-zero eigenvalues. $\boldsymbol{\Phi}_{MMSE}$ and $\boldsymbol{\Phi}_{ZF}$ are diagonal matrices for power allocation which are stated as $\boldsymbol{\Phi}^2_{MMSE} = (\mu^{-1/2}\boldsymbol{\Lambda}^{-1/2} - \boldsymbol{\Lambda}^{-1})_+$ and $\boldsymbol{\Phi}^2_{ZF} = P_0\boldsymbol{\Lambda}^{-1/2}/\mathrm{Tr}(\boldsymbol{\Lambda}^{-1/2})$, where μ is determined by P_0 and $(\cdot)_+$ means that the matrix has to be positive semi-definite.

3.2 HARQ-based Linear Precoding Scheme for Post-Combining

Since the optimal power allocation algorithms among different sub-channels were fully discussed for linear precoder structure, we could improve system performance such as MMSE and minimum bit error rate (BER). Hence, the precoding schemes could be applied to HARQ system to improve the performance.

The receiver combines all the received data of the L transmissions and the output data can be expressed as

$$\tilde{\mathbf{s}} = \frac{1}{L}\sum_{i=1}^{L}\tilde{\mathbf{s}}_{(i)}. \tag{2}$$

However, the above scheme simply adds the linear precoder to the conventional post-combining HARQ structure to optimize the performance for each separate transmission. In order to achieve more spatial diversity gains in MIMO-HARQ system, the modulated symbol vector $\tilde{\mathbf{s}}_{(i)}$ could be left-multiplied by a permutation matrix $\mathbf{P}_{(i)}$ before each transmission to fully utilize the strongest eigen-mode to transmit different symbols. Therefore, we could not only improve the performance for each transmission, but also average the reliabilities of all the symbols and thus the received signal can be expressed as

$$\tilde{\mathbf{s}}_{(i)} = \mathbf{G}_{(i)}\mathbf{H}_{(i)}\mathbf{F}_{(i)}\mathbf{P}_{(i)}\mathbf{s}_{(i)} + \mathbf{G}_{(i)}\mathbf{n}_{(i)}, \tag{3}$$

where for MMSE receiver and ZF receiver,

$$\mathbf{G}_{(i)} = (\mathbf{F}^H_{(i)}\mathbf{H}^H_{(i)}\mathbf{H}_{(i)}\mathbf{F}_{(i)}\mathbf{P}_{(i)} + \sigma_n^2\mathbf{P}_{(i)})^{-1}\mathbf{F}^H_{(i)}\mathbf{H}^H_{(i)}, \tag{4}$$

$$\mathbf{G}_{(i)} = (\mathbf{F}^H_{(i)}\mathbf{H}^H_{(i)}\mathbf{H}_{(i)}\mathbf{F}_{(i)}\mathbf{P}_{(i)})^{-1}\mathbf{F}^H_{(i)}\mathbf{H}^H_{(i)}. \tag{5}$$

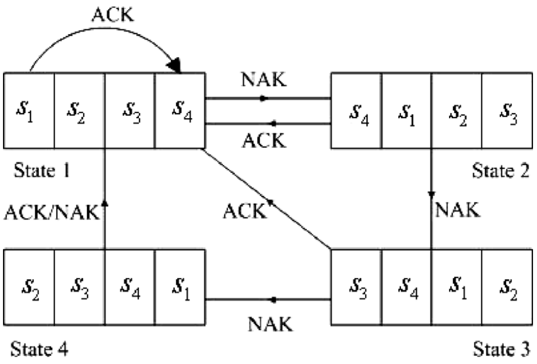

Fig. 1 State transition diagram for HARQ ($B = 4$, $L = 4$).

When $B = 4$ and $L = 4$, the transformation matrix can be stated as follows:

$$\mathbf{P}_{(1)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \ \mathbf{P}_{(2)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\mathbf{P}_{(3)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \ \mathbf{P}_{(4)} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

The process of data transmissions can be explained more clearly by using the following state transition diagram.

In Fig. 1, ACK and NAK are indicators for not to retransmit and retransmit during the next transmission period. The protocol consists of four states and s_1, s_2, s_3, s_4 are four modulated symbols in order. When there is no decoding error, the system would send ACK information to the transmitter and go back to state 1. Meanwhile, new data would be obtained from the data pool. Otherwise, next state will be entered and then the symbol vector would be left-multiplied by another permutation matrix. It is remarkable that the protocol will go back to State 1 from State 4 with both ACK/NAK and next data packet will be sent out.

4. Simulation Results and Performance Comparison

In this section, we illustrate the improvement in terms of average MSE and BER, that our proposed precoder-based scheme with sub-channel permutation provides over the original spatial multiplexing scheme [3] and simple additive linear precoding scheme without permutation matrix. Assuming that the channel changes slowly, $\mathbf{H}_{(i+1)}$ is the channel matrix at the $(i+1)$-th transmission with the distribution of mean $\rho\mathbf{H}_{(i)}$ and covariance matrix $\sigma_h^2(1-|\rho|^2)M\mathbf{I}_N$, where $\mathbf{H}_{(i)}$ is the CSI for the i-th transmission and ρ is the common time-correlation coefficient between $\mathbf{H}_{(i+1)}$ and $\mathbf{H}_{(i)}$ [6]. For Jakes model, the time-correlation $\rho = J_0(2\pi f_d T)$, where J_0 is the zero-th order Bessel function of the first kind, f_d is the

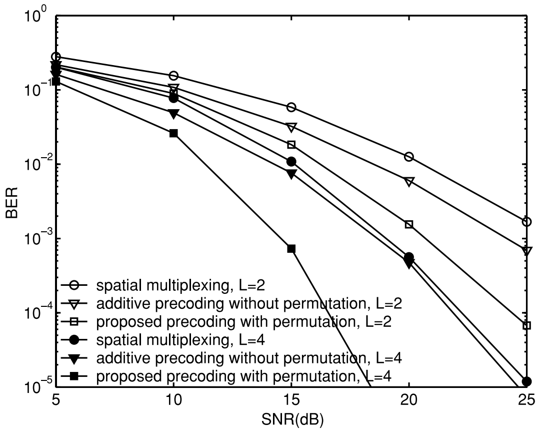

Fig. 2 BER comparison of three schemes based on post-combining HARQ systems.

Doppler frequency and T is the time interval between the two transmissions.

ZF receiver is adopted in the simulations and we further consider the case of a (4, 4) MIMO set-up with 16 QAM-modulated data streams. In simulations involving $f_d T$, a value of 0.3333 is used, this represents a Doppler frequency of 33.33 Hz (corresponding to a terminal velocity of 2 m/s for a carrier frequency of 5 GHz) and round trip delay T of 10 ms, so the time correlation coefficient ρ is 0.17. Channel coding is not adopted in the simulations. Signal to noise ratio (SNR) is defined as the ratio of the total transmit power across all transmit antennas to the receive noise variance on each receive antenna.

Figure 2 compares the BER of three methods. It can be seen that when the number of transmissions is relatively small (e.g. L=2), simple additive linear precoder improves the BER performance effectively, whereas it could not acquire obvious gains in comparison to spatial multiplexing scheme with the increase of L. The main reason is that additive linear precoder cannot average the spatial diversity gains among all the retransmission symbols and thus those symbols with errors cannot make fully use of the stronger sub-channels while correctly decoded symbols are always assigned to the strong ones without further benefits. Furthermore, with the use of the permutation matrix, our proposed scheme averages over retransmissions the variations in symbol reliabilities caused by the linear precoding and greatly decreases the BER. When BER is 10^{-4} and L is 4, our scheme obtains about 5.5 dB gains over spatial multiplexing scheme.

Figure 3 demonstrates average MSE of the sub-channels transmitting four symbols respectively for the above three schemes. It states that both spatial multiplexing scheme and the proposed scheme with sub-channel permutation guarantee similar MSE for all the symbols. On the other hand, the total MSE of the system decreases dramatically with the use of MMSE-based precoder in the proposed scheme in contrast to spatial multiplexing scheme. However, pure additive precoding scheme without permutation deteriorates the transmission quality of several symbols and leads to the performance degradation of the whole system.

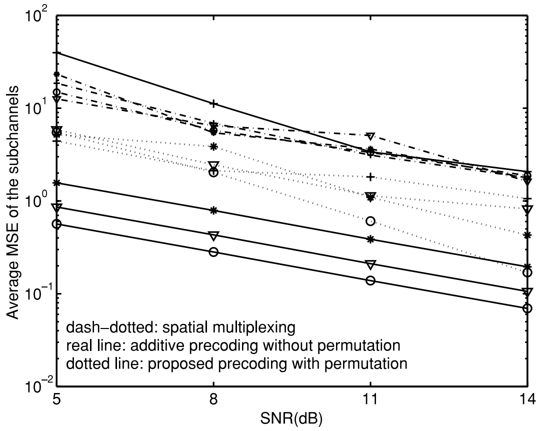

Fig. 3 Average MSE of the sub-channels used for the transmissions of four symbols (∘, +, ∗, ▽ represent four symbols respectively).

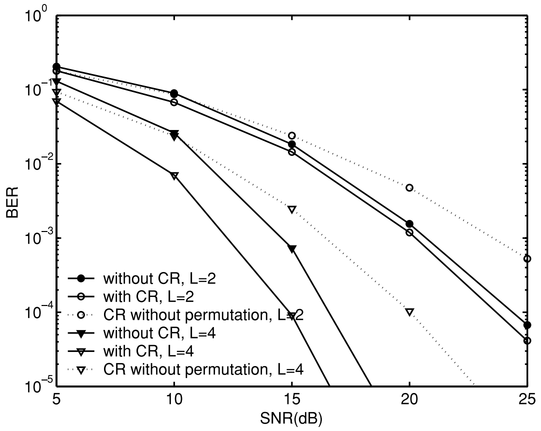

Fig. 4 BER performance of the proposed precoding scheme with permutation matrix and constellation rearrangement.

In order to illustrate the flexibility of the post-combining HARQ structure, we also compare the BER of the proposed scheme with or without combination of the constellation rearrangement (CR) [7] on bit-level. CR is a technique to achieve an identical averaging of the bit reliabilities by changing the mapping rules for the signal constellation. It shows in Fig. 4 that CR gets additional performance gains within the whole SNR regions and the gap between the two schemes becomes more obvious with the increase of the number of transmissions, which cannot be achieved in pre-combining HARQ strategy. The figure also shows the performance of the pure CR scheme without sub-channel permutation. It can be observed that the proposed linear precoding scheme with permutation achieves an additional diversity gains over the CR-based scheme.

5. Conclusions

In this paper, we have derived an effective linear precoding scheme for post-combining MIMO-HARQ system. The proposed linear precoder could be utilized to improve the performance for each transmission and achieve the joint spatial diversity gains with permutation matrices through several retransmissions. We have also shown that our approach not only decreases the average MSE of the system, but also diminishes the performance gap between different symbols and restrains the effects of the weaker sub-channel to a small region. On the other hand, post-combining scheme provides a flexible environment for linear precoding and constellation rearrangement methods. Hence, post-combining HARQ strategy is a very promising technology in wireless communications systems with multiple antennas.

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